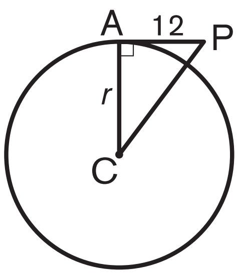
MATHCOUNTS® Minis **January 2020 Activity Solutions**

Warm-Up!

1. Recall that the diagonal of a square is also the hypotenuse of a 45-45-90 right triangle. Since a 45-45-90 right triangle with side length s, has a hypotenuse of length $s\sqrt{2}$, it follows that the diagonal, which is also the hypotenuse of one of the triangles, has length $\mathbf{20\sqrt{2}}$ cm. Alternatively, we can use the Pythagorean Theorem to determine the length of the hypotenuse c. We have $20^2 + 20^2 = c^2$, so $c = \sqrt{(400 + 400)} = \sqrt{800} = \mathbf{20\sqrt{2}}$ cm.

2. The area of rectangle ABCD is 27 × 11 = 297 units^2. If we subtract the area of the triangular region that is removed from the area of rectangle ABCD, the result is the area of pentagon ABEFD. Now CF = CD − FD = 27 − 15 = 12 units, and EC = BC − BE = 11 − 6 = 5 units. Thus the area of △CEF is 1/2 × 12 × 5 = 30 units^2. That means the area of pentagon ABEFD is 297 − 30 = **267** units^2.

3. In right triangle ABC, with $m\angle A = 30°$ and $m\angle C = 90°$, it follows that $m\angle B = 60°$, so △ABC is a 30-60-90 right triangle. Notice that right triangle BCD is also a 30-60-90 right triangle. We are told that CD = $\sqrt{3}$ cm, so by properties of 30-60-90 right triangles, we can conclude that BD = 1 cm, BC = 2 cm and AC = $2\sqrt{3}$ cm. Therefore, △ABC has area 1/2 × 2 × $2\sqrt{3}$ = $\mathbf{2\sqrt{3}}$ cm^2.

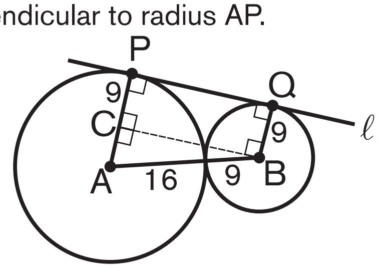


4. Since segment AP is tangent to the circle at A, segment PA will be perpendicular to radius AC. Because the area of the circle is 256π units^2, we can write the following equation and solve for r: $256\pi = \pi r^2 \rightarrow 256 = r^2 \rightarrow r = 16$ units. Using the Pythagorean Theorem with right triangle APC, we now can write the following equation and solve for PC: $PC^2 = 12^2 + 16^2 \rightarrow PC^2 = 144 + 256 \rightarrow PC^2 = 400 \rightarrow PC =$ **20** units.

The Problems are solved in the **MATHCOUNTS® Mini** video.

Follow-up Problems

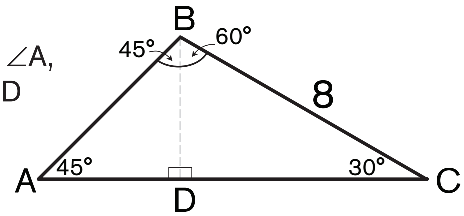
5. In the figure shown here, we have added the segment from B that is perpendicular to radius AP. This segment completes rectangle BCPQ, and now BQ = PC, so PC = 9 units. Radius AP is 16 units, so AC = 16 − 9 = 7 units. When we connect the two centers of the externally tangent circles, we get AB = 16 + 9 = 25 units. Now, using the Pythagorean Theorem with right triangle ABC, we have $25^2 = 7^2 + BC^2 \rightarrow 625 = 49 + BC^2 \rightarrow BC^2 = 576 \rightarrow BC = 24$ units. Because of rectangle BCPQ, we now know PQ = **24** units, too.



6. For right triangle ABC, you may recognize that AB = 15 = 3(5) and BC = 39 = 3(13) and conclude that this triangle has side lengths that are a derivative of the Pythagorean Triple 5-12-13, and AC = 3(12) = 36 units. Alternatively, using the Pythagorean Theorem, we get $15^2 + AC^2 = 39^2$, so $AC = \sqrt{(39^2 - 15^2)} = \sqrt{(1521 - 225)} = \sqrt{1296} = 36$ units. Since AC = AD + CD = 36 units, we have CD = 36 − AD. We are told that BD = CD + 9, so BD = 36 − AD + 9 = 45 − AD. Notice that △ABD is a right triangle with side lengths 15, AD, and 45 − AD, measured in units. Again using the Pythagorean Theorem, we get $15^2 + AD^2 = (45 - AD)^2$, so $225 + AD^2 = 2025 - 90 \times AD + AD^2$ and AD = (2025 − 225)/90 = 1800/90 = **20** units.

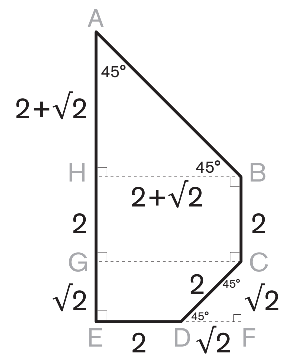
© 2020 MATHCOUNTS Foundation. All rights reserved. MATHCOUNTS Mini Solution Set

7. Consider △ABC, in which, $m\angle A = 45°$, and the side opposite ∠A, side BC, has length 8 units. If we draw a line from vertex B to a point D on side AC so that segment BD is perpendicular to side AC, △ABC is divided into two smaller triangles. As the figure shows, △ABD is a 45-45-90 right triangle, and △BCD is a 30-60-90 right triangle.



We are asked to determine the sum of the two missing side lengths of △ABC, AB + AC. Since △BCD is a 30-60-90 right triangle with hypotenuse BC of length 8 units, it follows that the shorter leg, side BD has length 4 units and the longer leg, side DC has length $4\sqrt{3}$ units. Now, since △ABD is a 45-45-90 right triangle with leg BD of length 4 units, it follows that leg AD also has length 4 units and hypotenuse AB has length $4\sqrt{2}$ units. We now know the missing side lengths for △ABC are AB = $4\sqrt{2}$ units and AC = $4 + 4\sqrt{3}$ units, so AB + AC = $\mathbf{4\sqrt{2} + 4 + 4\sqrt{3}}$ units.

8. If we draw segments BH and CG perpendicular to side AE, we create rectangle BCGH as shown. By properties of rectangles we know that HG = BC = 2 units. Then we can extend sides BC and ED to F to create 45-45-90 right triangle CDF as shown. Since CD = 2 units, by properties of 45-45-90 right triangles, we can conclude that CF = DF = $2/\sqrt{2} = \sqrt{2}$ units. It follows, then, that GE = $\sqrt{2}$ units and EF = GC = HB = $2 + \sqrt{2}$ units. Since △ABH is also a 45-45-90 right triangle, it follows that AH = HB = $2 + \sqrt{2}$ units. Finally, we see that AE = AH + HG + GE = $2 + \sqrt{2} + 2 + \sqrt{2} = 4 + 2\sqrt{2}$ units. Therefore, $a = 4$, $b = 2$ and $a + b = 4 + 2 =$ **6**.



© 2020 MATHCOUNTS Foundation. All rights reserved. MATHCOUNTS Mini Solution Set