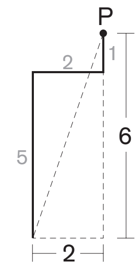
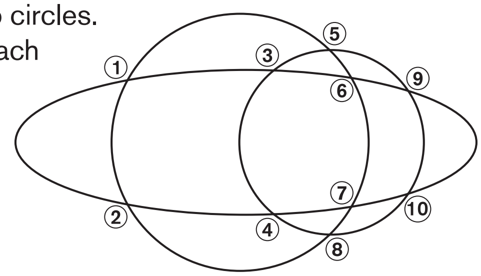
MATHCOUNTS® Minis December 2019 Activity Solutions

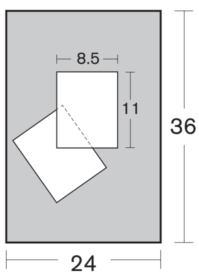
Warm-Up!



1. As the figure shows, Samantha walks a total of 5 + 1 = 6 mi north and 2 mi east. The distance we are looking for, from her starting point to P, is the length of the hypotenuse of a right triangle with legs of length 6 and 2. Using the Pythagorean Theorem, we have $d^2 = 6^2 + 2^2 \rightarrow d^2 = 36 + 4 \rightarrow d^2 = 40 \rightarrow d = \sqrt{40} \rightarrow d = \mathbf{2\sqrt{10}}$ miles.

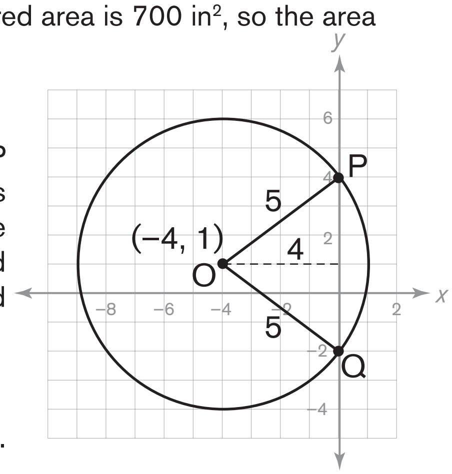
2. To maximize the number of points of intersection among the curves, we'll avoid scenarios in which a point is the intersection of all three curves. Let's determine the maximum possible number of points of intersection between the ellipse and each circle and between the two circles. The maximum possible number of points of intersection between each circle and the ellipse is 4 points, for a total of 4 + 4 = <u>8</u> points. The maximum possible number of points of intersection between the two circles is <u>2</u> points. The figure shows one way to draw two circles and an ellipse to achieve the maximum number of points at which at least two of the three curves intersect, which is 8 + 2 = **10** points.





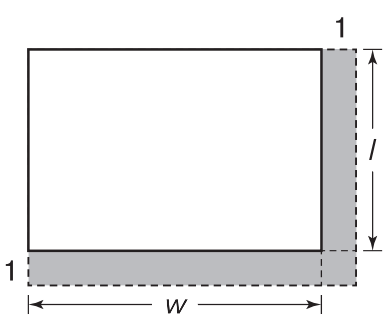
3. Since 1 ft = 12 inches, the tabletop has a total area of 24 × 36 = 864 in². Each sheet of paper has an area of 8.5 × 11 = 93.5 in². If there was no overlap between the two sheets of paper, there would be a total of 864 − (93.5 × 2) = 864 − 187 = 677 in² not covered by the sheets. We are told that the total uncovered area is 700 in², so the area of overlap must be 700 − 677 = **23** in².

4. The figure shows a circle with center O(−4, 1) and radii drawn to P and Q, the two points of intersection with the *y*-axis. Notice that isosceles triangle OPQ is composed of two 3-4-5 right triangles and that the points of intersection P and Q have coordinates (0, 1+3) = (0, 4) and (0, 1−3) = (0, −2), respectively. The sum of the *y*-coordinates of P and Q are 4 + (−2) = **2**.



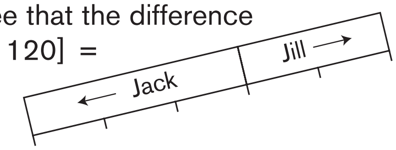
The Problems are solved in the MATHCOUNTS® Mini video.

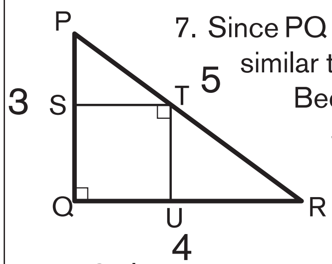
Follow-up Problems



5. Let the *l* and *w* represent the length and width of the original rectangle, respectively. From the figure, we see that when the length and width of the original rectangle are both increased by 1 inch, the resulting rectangle is composed of the unshaded $l \times w$ rectangle of area 108 in², and the three shaded rectangles with dimensions $1 \times w$, $l \times 1$ and 1×1. So, the resulting rectangle has area $108 + l + w + 1 = 109 + l + w$. We are told that the perimeter of the orignal rectangle is 42 inches. We have $2(l + w) = 42$, so $l + w = 21$. Therefore, the resulting rectangle has area 109 + 21 = **130** in².

6. As the figure shows, Jack travels down 3/5 of the hill in the same 2 minutes = 120 seconds that Jill travels up 2/5 of the hill. Using the formula rate = distance ÷ time, we see that the difference in Jack's and Jill's speeds is [(3/5 × 1260) ÷ 120] − [(2/5 × 1260) ÷ 120] = (1/5 × 1260) ÷ 120 = 252/120 = **2.1** ft/s.





7. Since PQ = PS + SQ and we are told that PQ = 3, we have 3 = PS + SQ → PS = 3 − SQ. For similar triangles PQR and PST, we can write the following proportion: 3/(3 − SQ) = 4/ST. Because QSTU is a square, it follows that SQ = QU = UT = ST. Substituting, we get 3/(3 − ST) = 4/ST. Cross-multiplying and solving, we see that 3(ST) = 4(3 − ST) → 3(ST) = 12 − 4(ST) → 7(ST) = 12 → ST = **12/7** units.

8. Let *a* represent the length of the shortest piece. The table shows the four ways to cut a 36-inch piece of rope into three pieces so that one piece is five inches longer than another, and one piece is twice as long as another.

	SHORT	MEDIUM	LONG
I	a	$a + 5$	$2a$
II	a	$2a$	$2a + 5$
III	a	$a + 5$	$2(a + 5)$
IV	a	$2a - 5$	$2a$

We can solve the following equations to determine the value of *a* and the length of the longest piece in each case.

For case I, $a + a + 5 + 2a = 36 \rightarrow 4a + 5 = 36 \rightarrow 4a = 31 \rightarrow a = 31/4$ inches, and the longest piece measures $2a = 2(31/4) = \underline{31/2}$ inches.

For case II, $a + 2a + 2a + 5 = 36 \rightarrow 5a + 5 = 36 \rightarrow 5a = 31 \rightarrow a = 31/5$ inches, and the longest piece measures $2a + 5 = 2(31/5) + 5 = \underline{87/5}$ inches.

For case III, $a + a + 5 + 2(a + 5) = 36 \rightarrow 4a + 15 = 36 \rightarrow 4a = 21 \rightarrow a = 21/4$ inches, and the longest piece measures $2(a + 5) = 2(21/4 + 5) = 82/4 = \underline{41/2}$ inches.

For case IV, $a + 2a - 5 + 2a = 36 \rightarrow 5a - 5 = 36 \rightarrow 5a = 41 \rightarrow a = 41/5$ inches, and the longest piece measures $2a = 2(41/5) = \underline{82/5}$ inches.

We must also consider the case in which the piece that is twice the length of another ALSO is five inches longer than that piece. In other words, when $2a = a + 5 \rightarrow a = 5$ inches. In this case, the longest piece has length 36 − (5 + 10) = <u>21</u> inches. The sum of all the possible lengths of the longest piece, then, is 31/2 + 87/5 + 41/2 + 82/5 + 21 = (155 + 174 + 205 + 164 + 210)/10 = 908/10 = **90.8** inches.