MATHCOUNTS® Minis October 2019 Activity Solutions

Warm-Up!

1. Each of the 16 team players is equally likely to be chosen as one of the 10 starters. So, the probability of any team player, including Joe, being randomly selected as a starter is 10/16 = **5/8**.

2. The box contains 7 blue, 3 red and 8 yellow bows, which is 18 bows altogether. Of the 18 bows in the box, 3 are red, meaning 15 are not red. Therefore, the probability that a bow, randomly selected from this box, is not red is 15/18 = **5/6**.

3. Let's first consider the probability that when flipping a fair coin 3 times the total number of heads (*H*) is greater than the total number of tails (*T*). The possible outcomes when flipping a coin 3 times are *HHH*, *HHT*, *HTH*, *THH*, *TTT*, *TTH*, *THT*, *HTT*. Notice that in half of these outcomes the total number of heads is greater than the total number of tails. This will be the case each time the coin is flipped an odd number of times. Therefore, the probability of the total number of heads being greater than the total number of tails when flipping a fair coin 37 times is **1/2**.

4. The total distance from 10 to −10 is 20 units. The distance from 7 to 10 is 3 units. Therefore, the chance that a randomly selected point from the interval $-10 \le x \le 10$ will be greater than or equal to 7 is **3/20**.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. Since we are only interested in the order in which Annika (A), Billy (B) and Catherine (C) are called, we can ignore the other 8 students in Ms. McGinn's class. The different orders in which the triplets can be called are ABC, ACB, BCA, BAC, CAB and CBA. In two of these cases, BCA and BAC, Billy is called first. Therefore, the probability that Billy is the first triplet called to Ms. McGinn's desk is 2/6 = **1/3**.

6. We're asked to determine the probability that two vertices chosen at random from the 8 vertices of a regular octagon are the endpoints of a side. First, notice that for each side there is exactly one pair of vertices that are endpoints of that side. Since a regular octagon has 8 sides, it follows that there are 8 outcomes for randomly choosing two vertices that happen to be endpoints of a side. Next, there are 8 outcomes for the first randomly chosen vertex, which leaves 7 outcomes for the second randomly chosen vertex. Although two different vertices can be randomly chosen in 2 different orders, the two vertices together represent a single outcome, regardless of the order chosen. That means there are $(8 \times 7) \div 2 = 56 \div 2 = 28$ total outcomes for randomly choosing two vertices of a regular octagon. Finally, the ratio of favorable outcomes to total outcomes, shows that the desired probability is 8/28 = **2/7**.

7. There are 16 squares in a 4-by-4 grid of unit squares. After the first coin is randomly placed in a unit square on the grid, there are 15 unit squares in which the second coin can randomly be placed. Of these 15 unit squares, 3 are in the row containing the first coin, and 3 are in the column containing the first coin. That means there are $15 - (3 + 3) = 15 - 6 = 9$ unit squares in which the second coin can randomly be placed so that the two coins do not lie in the same row or column. Thus, the probability that two randomly placed coins will not lie in the same row or column of unit squares is $9/15$ = **3/5**.

8. Think of the stick as a number line of length 1 unit, and let the point at which Shannon breaks the stick be x. The distance from 0 to point x is x. We are interested in the case when the distance x is more than twice the distance from x to 1, or when the distance from x to 1 is more than twice the distance from 0 to x. In other words, when $x \geq 2(1 - x)$ or $1 - x \geq 2x$ are true. Solving each inequality for x, we see that they are true when $x \geq 2/3$ or $x \leq 1/3$. That means Shannon is successful if the break point occurs within the first third of the stick or the last third. Thus, the probability that Shannon will break the stick such that the longer piece is more than twice the length of the shorter piece is $1/3 + 1/3$ = **2/3**.