MATHCOUNTS® Minis February 2019 Activity Solutions

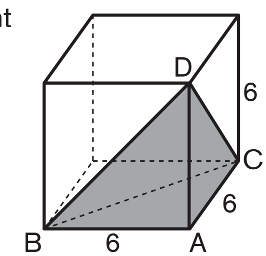
Warm-Up!

1. The total distance from 10 to −10 is 20 units. The distance from 7 to 10 is 3 units. Therefore, the chance that a randomly selected point from the interval $-10 \leq x \leq 10$ will be greater than or equal to 7 is **3/20**.

2. A point p on the number line will be closer to 4 than to 0 if $5 \geq p > 2$. The number line is 5 − 0 = 5 units in length. The portion of the number line to the right of 2 through 5 has length 5 − 2 = 3 units. Therefore, the probability that a randomly chosen point on this number line is closer to 4 than to 0 is 3/5 = 6/10 = **0.6**.

3. Let AB = 6 units. We know that AB = 3AD. So, 6 = 3AD and AD = 6/3 = 2 units. We also know that AB = 6BC. So, 6 = 6BC and BC = (1/6)AB = 6/6 = 1 unit. Finally, we know that AB = AD + CD + BC. So, 6 = 2 + CD + 1 and CD = 6 − 3 = 3 units. This mean the probability that a randomly selected point on segment AB is between C and D is 3/6 = **1/2**.

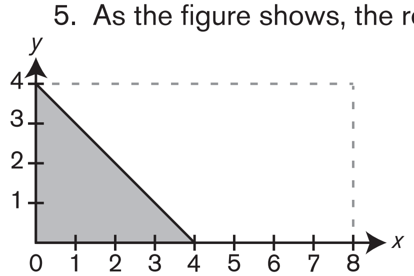
4. Tetrahedron ABCD is shown here. The volume of the tetrahedron (which is a pyramid with a triangular base) is $1/3 \times B \times h$, where B is the area of the base, and h is the height of the tetrahedron. The area of the base (ΔABC) is $1/2 \times 6 \times 6 = 18$ cm^2. Since the height is 6 cm, the volume of the tetrahedron is $1/3 \times 18 \times 6 =$ **36** cm^3.



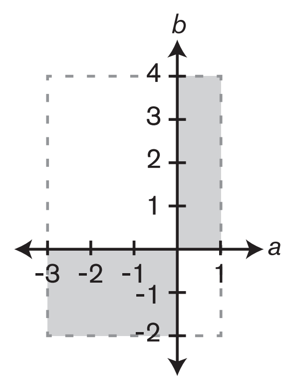
The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. As the figure shows, the region that includes all ordered pairs (x, y) such that $0 \leq x \leq 8$ and $0 \leq y \leq 4$ is a rectangle bounded by the lines $x = 0$ (the y-axis), $x = 8$, $y = 0$ (the x-axis) and $y = 4$, which has area $8 \times 4 = 32$ units^2. The shaded region includes all ordered pairs (x, y) where $x + y \leq 4$. This triangular region, which bounded by the lines $y = 0$ (the x-axis), $x = 0$ (the y-axis) and $y = 4 - x$, has area $1/2 \times 4 \times 4 = 8$ units^2. Thus, the probability that $x + y \leq 4$ is 8/32 = **1/4**.

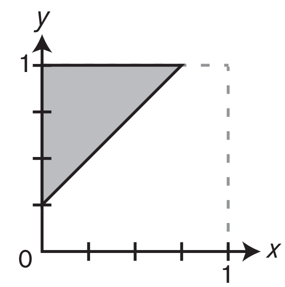


6. If we think of the possible values of a and b as perpendicular number lines, like the x- and y-axes of the coordinate plane, then we can solve this problem geometrically. As the figure shows, the region that includes all ordered pairs (a, b), where $-3 \leq a \leq 1$ and $-2 \leq b \leq 4$, is the rectangle bounded by the lines $a = -3$, $a = 1$, $b = -2$ and $b = 4$, which has area $4 \times 6 = 24$ units^2. The product ab is positive if and only if a and b are both positive or are both negative. The shaded regions include all such ordered pairs (a, b). These rectangular regions have a combined area of $1 \times 4 + 2 \times 3 = 4 + 6 = 10$ units^2. So, the probability that the product is positive is 10/24 = **5/12**.



7. Two numbers between 0 and 1 are chosen at random. We are asked to find the probability that the second number chosen will exceed the first number by at least $\frac{1}{4}$. If we choose the first number greater than $\frac{3}{4}$, the probability is 0 that the second number chosen could be greater than the first number by at least $\frac{1}{4}$. Therefore, the first number must be less than $\frac{3}{4}$. We must choose a number, x, between 0 and $\frac{3}{4}$. If we choose 0, then all numbers above $\frac{1}{4}$ will satisfy the requirement. That's $1-\frac{1}{4}=\frac{3}{4}$ of the available numbers. And if we choose $\frac{3}{4}$, then there is no probability for choosing the second number, i.e., the probability is 0. As the number chosen increases linearly from 0 to $\frac{3}{4}$, the probability decreases linearly from $\frac{3}{4}$ to 0. That makes an average probability of $\frac{1}{2}\times\frac{3}{4}=\frac{3}{8}$. There is a $\frac{3}{4}$ chance of choosing a number in the range of 0 to $\frac{3}{4}$ so the probability is $\frac{3}{4}\times\frac{3}{8}=\frac{9}{32}$.

Geometrically, we can represent all possible values of the two numbers as a square of area 1 unit^2. If the first number chosen is represented by values on the x-axis and the second number chosen is represented by values on the y-axis, then the shaded region includes all ordered pairs (x, y) such that $y > x$. This region is a right triangle bounded by the lines $y = 1$, $y = x + 1/4$ and $x = 0$ (the y-axis). The area of the shaded region is $1/2 \times 3/4 \times 3/4 = 9/32$ and the probability is $(9/32)/1 =$ **9/32**.



8. Triangle ABE is obtuse if and only if angle AEB is obtuse. This occurs if point E is inside a semicircle with diameter AB. Suppose the semicircle has a radius of 2 units. Then the area of the semicircle would be $1/2 \times \pi \times 2^2 = 2\pi$ units^2. Since the square has area $4 \times 4 = 16$ units^2, it follows that the probability the triangle is obtuse is $2\pi/16 = \pi/8 \approx$ **0.39**.