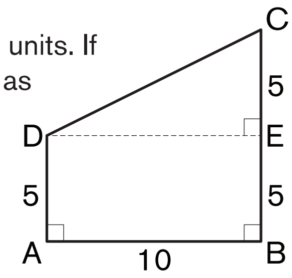
MATHCOUNTS® Minis November 2018 Activity Solutions

Warm-Up!

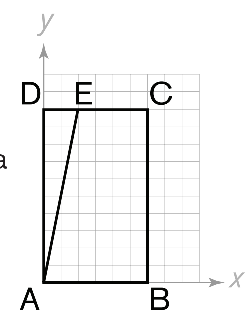
1. Since BA:AC = 3:2, it follows that AC = $\frac{2}{5}$BC. Therefore, AC = $\frac{2}{5}$ × 45 = **18**.

2. Since AD = 5 units and AB = BC = 2AD, it follows that AB = BC = 2(5) = 10 units. If we draw segment DE, parallel to side AB and intersecting segment BC at point E, as shown, rectangle ABED is formed with side lengths AD = BE = 5 units and AB = DE = 10 units, and with area is 10 × 5 = 50 units^2. So, right triangle DEC, with leg lengths DE = 10 units and CE = 10 − 5 = 5 units, has area (1/2) × 10 × 5 = 25 units^2. The total area of trapezoid ABCD, then, is 50 + 25 = **75** units^2.



3. Since segments MN and OP are parallel, we can conclude that $\triangle MNQ \sim \triangle POQ$ (Angle-Angle). Therefore, the ratios of corresponding sides of the triangles are congruent. Since ON = 24 units, it follows that OQ = 24 − NQ. We can set up the following proportion: NQ/(24 − NQ) = 12/20. Cross-multiplying and solving for NQ, we get 20(NQ) = 12(24 − NQ) → 20(NQ) = 288 − 12(NQ) → 32(NQ) = 288 → NQ = **9** units.

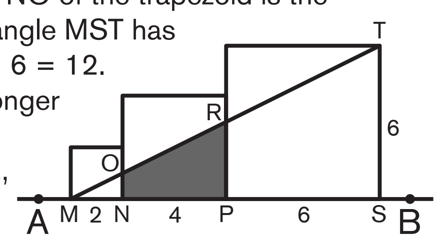
4. A segment drawn connecting A with E, as shown, creates a right triangle ADE and trapezoid ABCE. Triangle ADE has legs of length 10 units and 2 units, making its area (1/2) × 10 × 2 = 10 units^2. The area of trapezoid ABCE is difference between the area rectangle ABCD and the area of triangle ADE. Rectangle ABCD has area 10 × 6 = 60 units^2. So, the area of trapezoid ABCE is 60 − 10 = 50 units^2. The ratio of the area of triangle ADE to the area of trapezoid ABCE, then, is 10/50 = **1/5**.



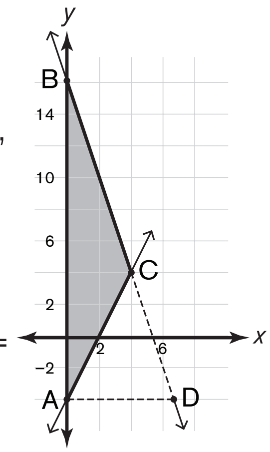
The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. We are asked to determine the area of the shaded quadrilateral, which happens to be a trapezoid. The height of the trapezoid is 4, the side length of the middle square. Notice that triangles MNO, MPR and MST, shown in the figure, are similar right triangles. The base NO of the trapezoid is the shorter leg of $\triangle MNO$ and the base PR is the shorter leg of $\triangle MPR$. Triangle MST has a shorter leg of length ST = 6 and a longer leg of length MS = 2 + 4 + 6 = 12. The ratio of the lengths of the shorter leg to the longer leg is 1:2. The longer leg of $\triangle MPR$ has length MP = 2 + 4 = 6, so its shorter leg must have length PR = 1/2 × 6 = 3. The longer leg of $\triangle MNO$ has length MN = 2, so its shorter leg must have length NO = 1/2 × 2 = 1. Therefore, the trapezoid has area 1/2 × (1 + 3) × 4 = **8** units^2.



6. Since $2(0) - 4 = -4$, it follows that $y = 2x - 4$ intersects the y-axis at A(0, −4). Similarly, since $-3(0) + 16 = 16$, it follows that $y = -3x + 16$ intersects the y-axis at B(0, 16). These two lines intersect each other when $2x - 4 = -3x + 16 \rightarrow 5x = 20 \rightarrow x = 4$ and $y = 2(4) - 4 = 8 - 4 = 4$, which we'll label C(4, 4). As the figure shows, the interior region formed by $y = 2x - 4$, $y = -3x + 16$ and the y-axis is △ABC. The dashed segments show the extension of $y = -3x + 16$ beyond point C and a portion of the horizontal line $y = -4$. The intersection of these dashed segements occurs when $-4 = -3x + 16 \rightarrow 3x = 20 \rightarrow x = 20/3$, at a point we'll label D(20/3, −4). As the figure shows, the area of △ABC is equal to the area of right triangle ABD minus the area of △ACD. Triangles ABD and ACD both have base length AD = $|20/3 - 0|$ = 20/3 units. Right triangle ABD has height $|16 - (-4)| = 20$ units, and △ACD has height $|4 - (-4)| = 8$ units. So, △ABC has area $(1/2)(20/3)(20) - (1/2)(20/3)(8) = (10/3)(20 - 8) = (10/3)(12) = (10)(4) =$ **40** units^2.



7. Triangle BCD is a 30-60-90 right triangle with a shorter leg of length 6. Based on properties of 30-60-90 right triangles, segment BC, the longer leg, has length $6\sqrt{3}$. Since M is the midpoint of segment AD, MD = $6\sqrt{3} \div 2 = 3\sqrt{3}$. For right triangle CDM, we know CD = 6 and DM = $3\sqrt{3}$, so we can use the Pythagorean Theorem to determine CM. We have $CM^2 = 6^2 + (3\sqrt{3})^2 \rightarrow CM = \sqrt{(36 + 27)} \rightarrow CM = \sqrt{(63)} \rightarrow CM = 3\sqrt{7}$. If $m\angle DBC = 30°$, then $m\angle BDA = 30°$ because they are alternate interior angles. Also $m\angle CKB = m\angle MKD$ since they are vertical angles. That means △CKB ~ △MKD, and BC/DM = CK/MK. Substituting and simplifying BC/DM, we have 2/1 = CK/MK, which means MK = ⅓CM → MK = ⅓ × $3\sqrt{7}$ → MK = $\mathbf{\sqrt{7}}$.

8. From the figure, we can see that the area of △ACD is the sum of the areas of △ABE and trapezoid BCDE. Also, we are told that the area of trapezoid BCDE is 8 times the area of △ABE. It follows that the area of △ACD is 9 times the area of △ABE. That means the ratio of sides BE and CD is $\sqrt{1}/\sqrt{9} = 1/3$. Since segments BE and CD are also sides of triangles EBX and CDX, respectively, it follows that the ratio of the areas of △EBX and △CDX is $1^2/3^2 = 1/9$. The problem states that the area of △CDX is 27 units^2, so the area of △EBX is (1/9) × 27 = 3 units^2. Using the method from the video, we can determine the areas of △BCX and △DEX by multiplying $\sqrt{3} \times \sqrt{27} = \sqrt{81} = 9$. Therefore, △BCX and △DEX each have an area of 9 units^2. We now can calculate the area of trapezoid BCDE to be 3 + 27 + 9 + 9 = 48 units^2. Using Harvey's trick results in the same answer since $(\sqrt{3} + \sqrt{27})^2 = (\sqrt{3} + 3\sqrt{3})^2 = (4\sqrt{3})^2 = 48$ units^2. So the area of △ABE is (1/8) × 48 = 6 units^2. Thus, the area of △ACD is 48 + 6 = **54** units^2. This also confirms our assertion that the area of △ACD is 9 times the area of △ABE since 9 × 6 = **54** units^2.