MATHCOUNTS® Minis December 2009 Activity Solutions

Warm-Up!

1. Five ordered pairs that satisfy this equation are (0, 1), (2, −1), (−1, 1/2), (3, −1/2) and (−1/2, 2/3). (Student answers may differ, as there are an infinite number of possible ordered pairs.)

2. The only ordered pairs of integers that satisfy this equation are (0, 1) and (2, −1).

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

3. Multiply the two equations together to get:

$$(x+\frac{1}{y})(y+\frac{1}{z})=1$$

$$xy+\frac{x}{z}+1+\frac{1}{yz}=1$$

Subtracting 1 from both sides of this new equation gives:

$$xy+\frac{x}{z}+\frac{1}{yz}=0$$

Multiply the above equation by z to get:

$$xyz+(x+\frac{1}{y})=0$$

We are given that $x+\frac{1}{y}=1$, so $xyz + 1 = 0$. Therefore, $xyz =$ **−1**.

4. Just as (1/2) × 2 = 1 on the right side of the equation in Problem 3, (1/3) × 3 = 1 as well. Therefore, multiplying these two equations together and following the same process as in Problem 3 will result in the same product, $xyz =$ **−1**.

5. As in Problems 3 and 4, (1/4) × 4 = 1, so multiplying these two equations together and following the same process will result in the same product, $xyz =$ **−1**.

6. Problems 3, 4 and 5 all have the same answer of −1. When multiplying the two provided equations together, as long as the product on the right side of the new equation is 1, then the pattern will continue and *xyz* will equal −1.

7. Multiply the two equations together to get:

$$(x+\frac{1}{y})(y+\frac{1}{z})=2$$

$$xy+\frac{x}{z}+1+\frac{1}{yz}=2$$

Subtracting 1 from both sides of this new equation gives:

$$xy+\frac{x}{z}+\frac{1}{yz}=1$$

Multiply the above equation by *z* to get:

$$xyz+(x+\frac{1}{y})=z$$

We are given that $x+\frac{1}{y}=2$, so $xyz + 2 = z$. Subtracting 2 from both sides of the equation gives $xyz = z - 2$, whose value changes with *z*. Thus, *xyz* does **not** have just one possible value.

8. We can rewrite $y+\frac{1}{z}=1$ as $y=1-\frac{1}{z}$ by subtracting $\frac{1}{z}$ from both sides of the equation. So:

$$x+\frac{1}{y}=x+\frac{1}{1-\frac{1}{z}}=x+\frac{z}{z-1}=1$$

Subtracting *x* from both sides of the equation, then multiplying both sides by $z - 1$ gives:

$$\frac{z}{z-1}=1-x$$

$$z=(1-x)(z-1)$$

$$z=z-1-xz+x$$

Subtract *z* and *x* from both sides of the equation, then add 1 to both sides to get $xz = x - 1$. Finally, divide both sides of the equation by *x*:

$$z=\frac{x-1}{x}=1-\frac{1}{x}$$

$$z+\frac{1}{x}=1$$

Therefore, **yes**, $z+\frac{1}{x}$ must also equal 1.