Warm-Up!

1. Five ordered pairs that satisfy this equation are (0, 1), (2, −1), (−1, 1/2), (3, −1/2) and (−1/2, 2/3). (Student answers may differ, as there are an infinite number of possible ordered pairs.)

2. The only ordered pairs of integers that satisfy this equation are (0, 1) and (2, −1).

The Problems are solved in the MATHCOUNTS Mini video.

Follow-up Problems

3. Multiply the two equations together to get:

\[(x + \frac{1}{y})(y + \frac{1}{z}) = 1\]

\[xy + \frac{x}{z} + 1 + \frac{1}{yz} = 1\]

Subtracting 1 from both sides of this new equation gives:

\[xy + \frac{x}{z} + \frac{1}{yz} = 0\]

Multiply the above equation by z to get:

\[xyz + (x + \frac{1}{y}) = 0\]

We are given that \(x + \frac{1}{y} = 1\), so \(xyz + 1 = 0\). Therefore, \(xyz = -1\).

4. Just as \((1/2) \times 2 = 1\) on the right side of the equation in Problem 3, \((1/3) \times 3 = 1\) as well. Therefore, multiplying these two equations together and following the same process as in Problem 3 will result in the same product, \(xyz = -1\).

5. As in Problems 3 and 4, \((1/4) \times 4 = 1\), so multiplying these two equations together and following the same process will result in the same product, \(xyz = -1\).
6. Problems 3, 4 and 5 all have the same answer of −1. When multiplying the two provided equations together, as long as the product on the right side of the new equation is 1, then the pattern will continue and $xyz$ will equal −1.

7. Multiply the two equations together to get:

$$ (x + \frac{1}{y})(y + \frac{1}{z}) = 2 $$

$$ xy + \frac{x}{z} + 1 + \frac{1}{yz} = 2 $$

Subtracting 1 from both sides of this new equation gives:

$$ xy + \frac{x}{z} + \frac{1}{yz} = 1 $$

Multiply the above equation by $z$ to get:

$$ xyz + (x + \frac{1}{y}) = z $$

We are given that $x + \frac{1}{y} = 2$, so $xyz + 2 = z$. Subtracting 2 from both sides of the equation gives $xyz = z - 2$, whose value changes with $z$. Thus, $xyz$ does not have just one possible value.

8. We can rewrite $y + \frac{1}{z} = 1$ as $y = 1 - \frac{1}{z}$ by subtracting $\frac{1}{z}$ from both sides of the equation. So:

$$ x + \frac{1}{y} = x + \frac{1}{\frac{1}{z}} = x + \frac{z}{z-1} = 1 $$

Subtracting $x$ from both sides of the equation, then multiplying both sides by $z - 1$ gives:

$$ \frac{z}{z-1} = 1 - x $$

$$ z = (1 - x)(z - 1) $$

$$ z = z - 1 - xz + x $$

Subtract $z$ and $x$ from both sides of the equation, then add 1 to both sides to get $xz = x - 1$. Finally, divide both sides of the equation by $x$:

$$ z = \frac{x-1}{x} = 1 - \frac{1}{x} $$

$$ z + \frac{1}{x} = 1 $$

Therefore, yes, $z + \frac{1}{x}$ must also equal 1.