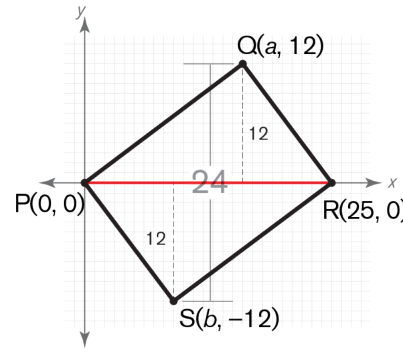
Errata for the 2015-2016 through 2019-2020 School Handbooks

2019-2020 MATHCOUNTS School Handbook (last updated on September 18, 2019)

Workout 8 Problem #243: The correct answer is **300** units^2. The updated solution follows:

Rectangle PQRS has diagonals PR and QS. The problem states that the *y*-coordinates of P and R are equal and that the *x*-coordinate of R is 25 more than the *x*-coordinate of P. If we let P(0, 0) be a vertex of the rectangle, then so is R(25, 0). As the figure shows, diagonal PR divides rectangle PQRS into congruent right triangles, PQR and PSR, that share a base of length 25. Since we are told that the *y*-coordinates of Q and S differ by 24, it follows that the height of each triangle must be half that amount, or 12. We now see that rectangle PQRS has area 2 × (1/2) × 25 ×12 = **300** units^2.



NOTE:	*The above error appears ONLY in digital downloads of the* 2019-2020 MATHCOUNTS School Handbook-**Part 2** *downloaded prior to 9/20/19.*

2018-2019 MATHCOUNTS School Handbook (last updated on September 15, 2018)

We do not have records of errata for this School Handbook.

2017-2018 MATHCOUNTS School Handbook (last updated on November 13, 2017)

Clarification for Travel Stretch Problem #29: As stated in the second sentence, Ansel travels against the current on his return trip. It may not be clear, however, that the average speed of 20 mi/h relative to the water applies to Ansel's entire trip.

Alternate Answer Warm-Up 14 Problem #161: The solution that results in the given answer of 30 assumes that *k* is an integer. However, this is not explicitly stated in the problem. The final sentence should read, "… what is the least possible **integer** value of *k*?" As written, another answer is acceptable. If we consider the three consecutive integers to be 1, 2 and 3, they have a product of 6. Therefore, solving 6 = 22 × 14 × *k*, for *k*, we get 6 = 308*k* and *k* = 6/308 = **3/154**, which is the least possible value of *k*.

Clarification for Workout 2 Problem #190: The container with the given shape and dimensions has a volume of $\pi \times 3^2 \times 12 = 108\pi$ in^3, which is equivalent to approximately $108\pi\,(1/1.8) \approx$ 188 fluid ounces. The problem, however, states that this container "holds 20 fluid ounces," implying that the inner dimensions of the cylindrical container are considerably less than its outer dimensions. In addition, the question asks, "how many fluid ounces will a similar container with a radius of 4.5 inches hold?" The phrase "similar container" may be interpreted to mean another cylinder like the original one, with a radius of 4.5 inches and a height of 12 inches. It should be interpreted to mean a similar cylinder in which the ratios of corresponding dimensions are equal.

Competition Coach Toolkit: There is a typo at the bottom of p. 46 in the box containing the formulas for the Sum and Difference of Cubes. The formulas are reversed. See correction below:

Sum and Difference of Cubes

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

NOTE: *The above error appears ONLY in the print version of the* 2017-2018 MATHCOUNTS School Handbook *and in any digital copies downloaded prior to 10/4/17.*

2016-2017 MATHCOUNTS School Handbook (last updated on December 7, 2016)

Warm-Up 3 Problem #25: The answer of 9 is correct; however, the solution assumes that "the larger number is 4 times the smaller number", meaning $4x > x$. Since this is not explicitly stated in the problem, the solution also should address the case in which the smaller number is 4 times the larger, or $4x < x$. In this case, we still have $y = 4x$, so $x + y = 3(4x) + 18$. Substituting $4x$ for y and solving for x, yields $x + 4x = 3(4x) + 18 \rightarrow 5x = 12x + 18 \rightarrow -7x = 18 \rightarrow x = -18/7$, which means that $y = 4(-18/7) = -72/7$. Since these are not integer values, it follows that Esme's numbers are 9 and 36, the smaller of which is **9**.

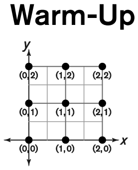
Workout 8 Problem #185: There is a typo in the printed solution for this problem. The first sentence should read as follows:

"The expression $(n^2 - 9)/(n^2 - 4)$ can be factored to get $((n - 3)(n + 3))/((n - 2)(n + 2))$."

In addition, there is another answer that should be considered. One could argue that the problem should be limited to only "meaningful" expressions, in which case, when $n = 2$, the expression $-5/0$ is meaningless. So, although the expression has a numerator and denominator with a GCF of 5, which is greater than 1, this expression when $n = 2$ should be excluded since division by 0 is not permitted. With this reasoning, **805** would be the correct the answer. This problem provides a great opportunity to discuss division by 0 and the greatest common divisor (or greatest common factor) of 0 and any other number.

2015-2016 MATHCOUNTS School Handbook (last updated on November 12, 2015)

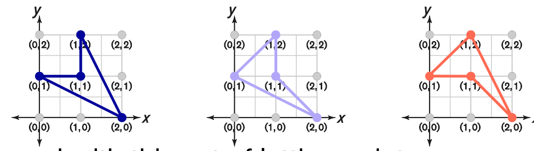
Warm-Up 9 Problem #145: The correct answer is **94** quadrilaterals. The updated solution follows:



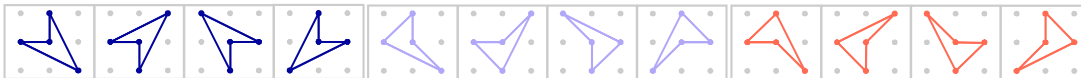
The coordinate grid shown here has the nine lattice points with integer coordinates (x, y) such that $0 \leq x \leq 2$ and $0 \leq y \leq 2$. There are ${}_9C_4 = 9!/(5! \times 4!) = 126$ ways to choose 4 of these 9 lattice points.

However, not all of these combinations can represent vertices of quadrilaterals. In fact, any combination that includes three collinear lattice points from one of the 3 rows, one of the 3 columns or one of the 2 diagonals won't work. For each of the $3 + 3 + 2 = 8$ sets of three collinear lattice points there are 6 different ways to choose the fourth lattice point. That's a total of $8 \times 6 = 48$ combinations of 4 lattice points that won't work. We subtract these from our original calculation to get a total of $126 - 48 = 78$ ways to choose 4 lattice points that will form a quadrilateral.

The points (0, 1), (1, 1), (1, 2), (2, 0) give us these three quadrilaterals, each of which can be rotated 90°, 180° and 270° resulting in a total of 9 additional quadrilaterals.

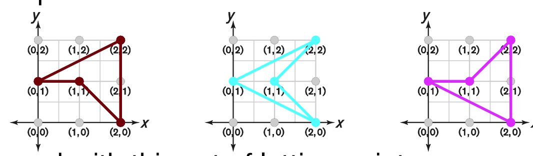


The figure below shows all 12 quadrilaterals that can be formed with this set of lattice points.

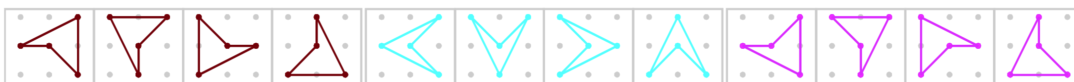


The 78 ways we calculated to choose 4 lattice points includes one of these three quadrilaterals and its three rotations (4 quadrilaterals total). So that means we need to account for the remaining $12 - 4 = 8$ quadrilaterals. We've now counted $78 + 8 = 86$ quadrilaterals.

The points (0, 1), (1, 1), (2, 0), (2, 2) also give us three quadrilaterals, each of which can be rotated 90°, 180° and 270° resulting in a total of 9 additional quadrilaterals.



The figure below shows all 12 quadrilaterals that can be formed with this set of lattice points.



Again, we've already counted one of these quadrilaterals and its three rotations, but we need to account for the remaining 8 quadrilaterals. That brings our total to $86 + 8 =$ **94** quadrilaterals.

The published solution for Warm-Up 9 Problem #145 explains how to solve a problem that asks:

> *How many different sets of four vertices that form a quadrilateral have integer coordinates (x, y) such that $0 \le x \le 2$ and $0 \le y \le 2$?*

We have demonstrated, as does the published solution, that there are 78 such sets of vertices.

NOTE: *The following errata appear ONLY in the print version of the* 2015-2016 MATHCOUNTS School Handbook *and in any digital copies downloaded prior to 9/9/15.*

Warm-Up 7 Problem #104: There is a typo in the last line: "between the **mean**" should be "between the **means**".

Warm-Up 12 Problem #199: There is a typo in the first line: "to" should be "two".

Warm-Up 14 Problem #215: There is a typo in the second line: "But" should be "If".

Modular Arithmetic Stretch Problem #248: As written, there is nothing that restricts the answer from being 6, as 6 ÷ 7 = 0 R6 and 6 ÷ 11 = 0 R6. Therefore, the problem should read:

What is the least **integer greater than 6** that leaves a remainder of 6 when it is divided by 7 and by 11?