Proofigami

Use origami to prove the Pythagorean Theorem.

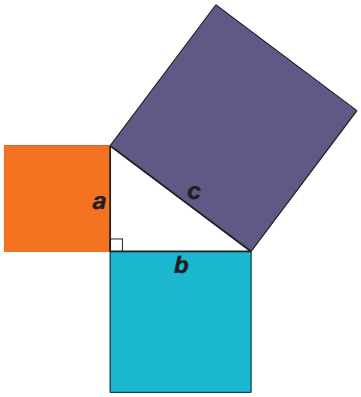
The Pythagorean Theorem is commonly introduced to students in middle school, and understanding why this theorem works will help your students apply it in their problem solving—during middle school geometry and beyond. This activity gives students a brief introduction to the Pythagorean Theorem and then guides them through what we call Proofigami. This fun activity will feel a lot like origami and will give Club Leaders a visual and tactile tool that makes believing this Theorem easier.

MATERIALS NEEDED

- Origami paper or regular paper cut into a square (1 sheet per student)
- Diagram Instructions Handout (optional–available at www.mathcounts.org/proofigami)
- Colored pencils, markers or crayons (3 colors)
- Proofigami Set-up Instructions PPT presentation (optional—available at www.mathcounts.org/proofigami)
- Proofigami Instructional Video (optional–available at www.mathcounts.org/proofigami)
- Scissors (optional)

INTRODUCTION TO THE PYTHAGOREAN THEOREM

The Pythagorean Theorem is a relation among the three sides of a right triangle. It states that the sum of the squares of the lengths of the two sides adjacent to the right angle, sides labeled *a* and *b* in the diagram below, is equal to the square of the hypotenuse, opposite the right angle, labeled *c*. It is commonly written as $a^2 + b^2 = c^2$, with *a*, *b* and *c* representing the side lengths. The theorem is named after the Greek mathematician Pythagoras.



SETTING UP PROOFIGAMI

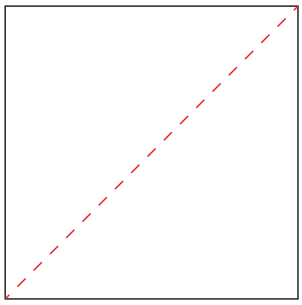
The following pages provide the instructions with corresponding figures. We suggest reading through the instructions yourself before presenting this to your club members, to make sure you understand all of the steps. It also may be helpful to watch the Proofigami Instructional Video, available at www.mathcounts.org/proofigami.

In addition to the Club Leader instructions that follow, we have created the Proofigami Set-up Instructions PPT presentation so your students can follow along with you as you set up your Proofigami creations. It also will be helpful to give each student (or each pair of students) a copy of the Diagram Instructions Handout. Origami paper is best if accessible, but students can also take regular paper and cut it into a square. Only one piece per student is required, but in case your students need to redo their work or want to practice with regular paper first, you might want a few extra available.

Proofigami Set-up Instructions

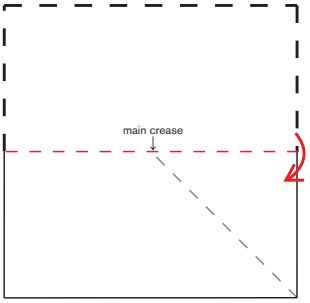
Step 1: Fold the origami paper along a diagonal and then open it back up. Position your paper so that the diagonal crease starts at the bottom left corner and slopes up to the top right corner, as shown in Fig. 1.

Fig. 1



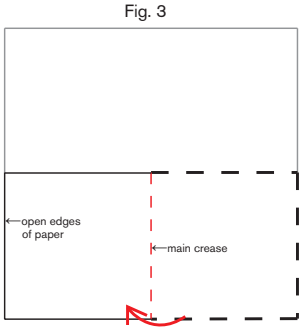
Step 2: Fold the paper in half by bringing the top edge of the paper down to meet the bottom edge of the paper.

Fig. 2



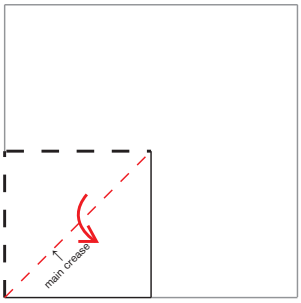
Step 3: Keep the paper folded in half from step 2. Fold the right edge of the paper over to meet the left edge. This should create a square that is one-quarter of the size of your original paper. The main crease should be on the right edge of the paper, with the open edges of the paper on the left and bottom. You should not see any fold impressions on the front of this smaller square you've created (but there will be one diagonal fold impression on the back).

Fig. 3



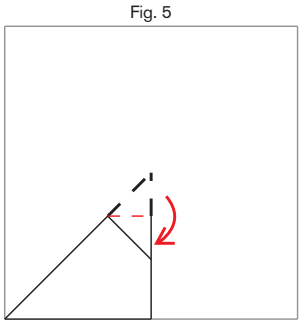
Step 4: Keep the paper folded from step 3. Fold along the diagonal to create a triangle. To do this, take the top left corner and fold it down to meet the bottom right corner.

Fig. 4



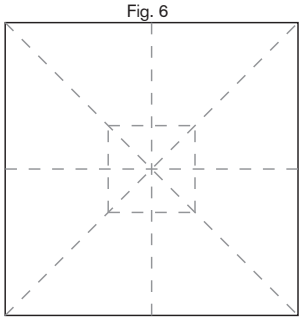
Step 5: Keep the paper folded from step 4. Take the top vertex and fold down so that a small triangle with two sides of equal length is created at the top (students' triangles will be of varying sizes).

Fig. 5



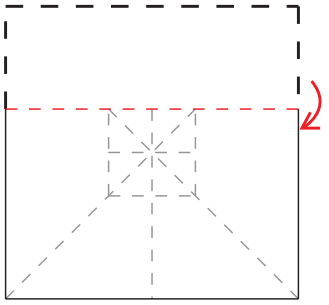
Step 6: Completely unfold your paper back to its original size and shape. Orient the paper as shown and make sure you can see all of the creases, indicated by the dashed lines. You should see: a smaller square, formed by the creases, in the center of the larger square paper; two diagonal creases across the square paper; and a horizontal and vertical crease dividing the square paper into quarters.

Fig. 6



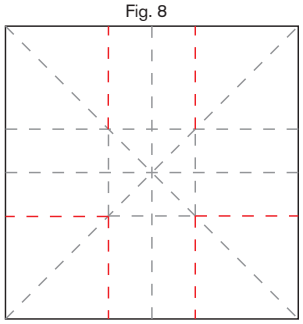
Step 7: Fold the top edge down so that a new crease along the top side of the small inside square is created and now extends across the entire length of the paper, from left to right. Unfold your paper and then repeat this step for the right, bottom and left sides of the small inside square.

Fig. 7



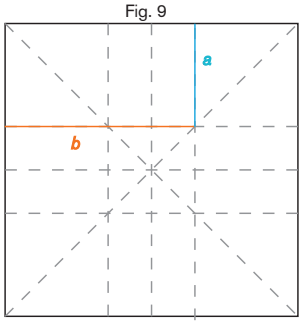
Step 8: Unfold your square and make sure you have all of the creases shown. You should see that all the sides of the small interior square have now been extended to meet the edges of the paper.

Fig. 8



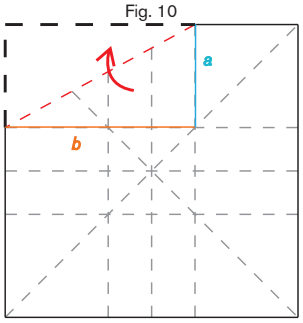
Step 9: With your paper still unfolded, use two different colored pencils to label the two creases shown colored in Fig. 9. The *b* leg should be a segment of the uppermost horizontal crease, starting at the left edge of the paper and extending to the rightmost vertical crease. The *a* leg should be a segment of the rightmost vertical crease, starting at the top of the paper and extending down to meet leg *b*.

Fig. 9



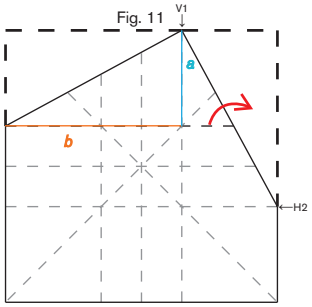
Step 10: Take the top left corner of your paper and fold it back to create what will be the hypotenuse, leg *c*, which will connect legs *a* and *b*. It may be helpful to flip your paper over so you can still fold down for this step.

Fig. 10



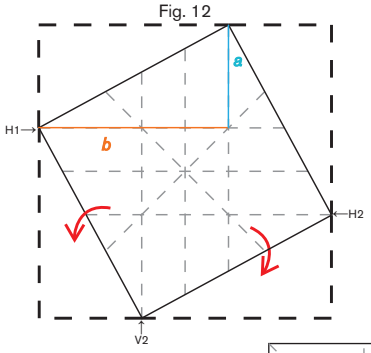
Step 11: Locate the rightmost point of the lowest horizontal crease, labeled H2 in Fig. 11. Locate the top point of the rightmost vertical crease, labeled V1 in Fig. 11. Repeat what you did in step 10 so you create a hypotenuse that connects the point H2 to the point V1.

Fig. 11



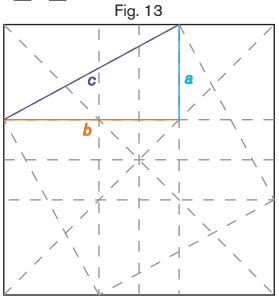
Step 12: Locate the bottom point of the leftmost vertical crease, labeled V2 in Fig. 12. Repeat what you did in steps 10 and 11, so you create a hypotenuse that connects the point H2 to the point V2. Repeat this one more time to connect the point V2 to the point H1.

Fig. 12



Step 13: Unfold your paper completely. Use a third colored pencil to label the hypotenuse *c*, which connects legs *a* and *b*, and label it *c*. Finally, cut or gently tear along the side labeled *a*. Cut or tear *only* the length of leg *a*, no farther.

Fig. 13



Now that you've created your Proofigami, and you can use it to prove the Pythagorean Theorem!

PROVING THE PYTHAGOREAN THEOREM

First, let's label two additional segments with the lengths of *a* and *b*, as shown in Fig. 14. (You can measure to convince yourself that the segments of length *a* are equal and the segments of length *b* are equal.)

We are going to fold back four triangles of equal area. These four triangles are numbered in Fig. 15, and all have side lengths *a*, *b* and *c*, so they must be congruent (exactly the same). Fold back triangles 1 and 2, and then fold back triangles 3 and 4. By eliminating triangles 1, 2, 3 and 4, we are left with two squares of area a^2 and b^2, which is a total area of $a^2 + b^2$.

To prove that this $a^2 + b^2$ is equal to c^2, we should be able to start with our full sheet of paper, again eliminate four triangles of sides *a*, *b* and *c*, and be left with a square of side length *c* (and area of c^2).

Unfold back to your original square paper. Fold back the four triangles, labeled 1, 4, 5 and 6, shown in Fig. 16. These are congruent to the four triangles you folded back in steps 10-12. The remaining area is a square with side lengths equal to *c* and area equal to c^2.

Thus, we have shown that a total area of $a^2 + b^2$ is the same as the area of c^2, so $a^2 + b^2 = c^2$. If your students aren't convinced, measure your *a*, *b* and *c* lengths and plug them into the Pythagorean Theorem to show the equality!

Fig. 14

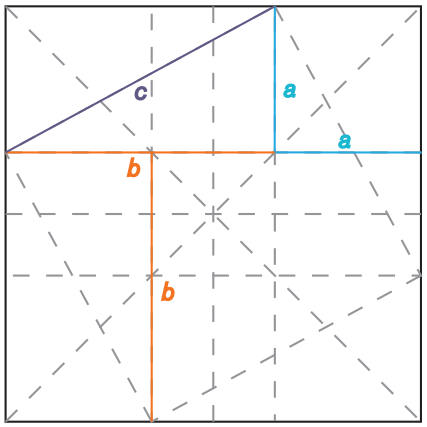


Fig. 15

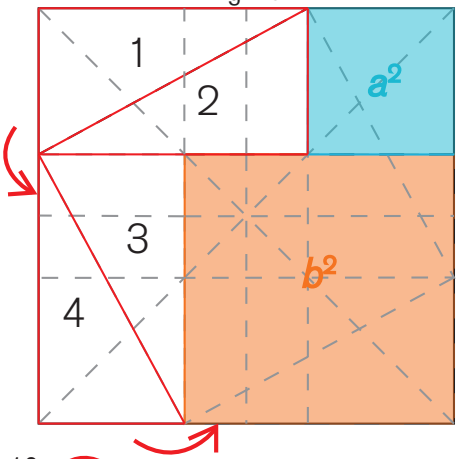
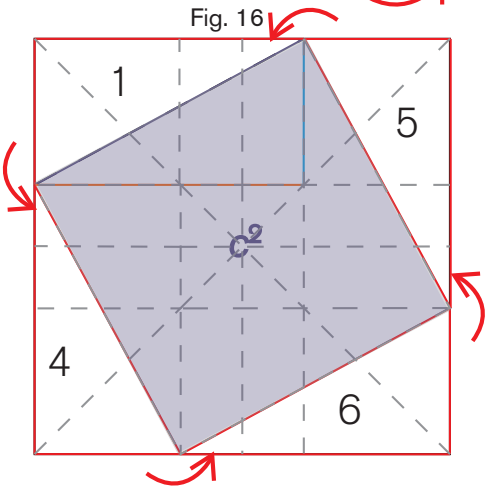


Fig. 16



DO MORE WITH THIS ACTIVITY

We presented a proof for the Pythagorean Theorem in this activity, but there are other origami proofs out there. Look into a couple of the following proofs:

- Dividing lines and angles into two or more equal parts
- Irrationality of the square root of 2
- Drawing perpendiculars and parallels to straight lines