THE NATIONAL MATHCLUB

Piggy Bank Day

The difficulty level of each problem is indicated by the value that is written before it.

1¢ 1. Using pennies, nickels, dimes and quarters, what is the least number of coins needed to make 68 cents in change?

2¢ 2. A piggy bank contains only nickels, dimes and quarters. There is at least one of each type of coin in the bank. If the total value of the coins in the piggy bank equals 60 cents, how many quarters are in the bank?

3¢ 3. Sasha has $3.20 in U.S. coins. She has the same number of quarters and nickels. What is the greatest number of quarters she could have?

3¢ 4. Krista put 1 cent into her new bank on a Sunday morning. On Monday, she put 2 cents into her bank. On Tuesday, she put 4 cents into her bank, and she continued to double the amount of money she put into her bank each day for two weeks. On what day of the week did the total amount of money in her bank first exceed $2?

5¢ 5. Anjali has 230 quarters, 300 dimes and 165 nickels. Niki has 210 quarters, 316 dimes and 173 nickes. How many more cents than Niki does Anjali have?

5¢ 6. Karla has only dimes and quarters in her piggy bank. The value of her dimes is exactly five times the value of her quarters. What is the least number of coins she could have in her piggy bank if there is at least one quarter?

10¢ 7. In how many different ways can 30 cents be made from any combination of quarters, dimes, nickels and/or pennies?

10¢ 8. Susan has exactly \$1.50 in change. She has 30 total coins (pennies, nickels, dimes and/or quarters). Half of the coins are nickels. There are two-thirds as many pennies as there are nickels. How many dimes does Susan have?

10¢ 9. A coin machine keeps 8.9 cents per \$1 of coins inserted. This ratio also is maintained when only a fraction of a dollar is inserted. If Ashley inserts \$50.50 worth of change into the machine, how much money will the machine give her back?

10¢ 10. My piggy bank has only nickels, dimes and dollar bills. The ratio of nickels to dimes is 2:3, and the ratio of dimes to dollar bills is 10:1. What is the ratio of coins to dollar bills? Express your answer in the form *a:b*, where *a* and *b* are positive integers with no common factors greater than 1.

10¢ 11. There are equal numbers of pennies, nickels, dimes and quarters in a piggy bank. Two coins are pulled out, one at a time, and each coin is replaced before the next is drawn. What is the probability that the sum of the values of the two coins will be less than 15 cents? Express your answer as a common fraction.

15¢ 12. John's piggy bank had \$1.20 when he went to bed on Monday. On Tuesday morning, he put three coins in the bank. He put in three more coins in the afternoon and then three more in the evening. That night, there was a total of \$2.20 in his bank. If no coin is worth more than 25 cents, what is the greatest amount, in cents, John could have put in the bank on Tuesday evening?

15¢ 13. Jennifer, Mike and Carol each have a bunch of quarters. Jennifer and Mike have 26 quarters together. Jennifer and Carol have 20 quarters together. Mike and Carol have 22 quarters together. How many cents does Mike have?

Club Leader Solution Guide!

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Piggy Bank Day

1. Using pennies, nickels, dimes and quarters, what is the least number of coins needed to make 68 cents in change? **1¢**

   To use the least number of coins, use as many of the highest-value coins (the quarters) as possible. Two quarters is the most possible without going over 68 cents. Continue this process with the next coin of highest-value, the dime, then the nickel, then the penny. Find that the least number of coins needed to make 68 cents is 7 coins—2 quarters, 1 dime, 1 nickel, and 3 pennies.

2. A piggy bank contains only nickels, dimes and quarters. There is at least one of each type of coin in the bank. If the total value of the coins in the piggy bank equals 60 cents, how many quarters are in the bank? **2¢**

   With at least one of each type of coin in the bank, there are already 40 cents in the bank. Adding another quarter would give more than 60 cents, so there is only 1 quarter.

3. Sasha has $3.20 in U.S. coins. She has the same number of quarters and nickels. What is the greatest number of quarters she could have? **3¢**

   Represent this scenario with an equation. $0.25n + 0.05n = 3.20$, where n = a positive value representing the amount of each type of coin. Because there needs to be the same number of quarters and nickels, there is only one variable for both. This simplifies to $0.30n = 3.20$. Using the inverse operation, find that $n = 3.20 / 0.30 = 10\ 2/3$. Because there cannot be a fraction of a coin, 10 is the largest number where there could be the same number of nickels and quarters. So, Sasha could have 10 quarters.

4. Krista put 1 cent into her new bank on a Sunday morning. On Monday, she put 2 cents into her bank. On Tuesday, she put 4 cents into her bank, and she continued to double the amount of money she put into her bank each day for two weeks. On what day of the week did the total amount of money in her bank first exceed $2? **3¢**

   Continuing to add doubled amounts of money each day, find that Krista will jump from $1.26 to $2.54 on Sunday.

5. Anjali has 230 quarters, 300 dimes and 165 nickels. Niki has 210 quarters, 316 dimes and 173 nickes. How many more cents than Niki does Anjali have? **5¢**

   Anjali has $(230 \times 0.25) + (300 \times 0.10) + (165 \times 0.05) = \$95.75$.
   Niki has $(210 \times 0.25) + (316 \times 0.10) + (173 \times 0.05) = \$92.75$.
   Anjali has $\$95.75 - \$92.75 = \$3$ more than Niki, which is $(3 \times 100) = 300$ cents.

6. Karla has only dimes and quarters in her piggy bank. The value of her dimes is exactly five times the value of her quarters. What is the least number of coins she could have in her piggy bank if there is at least one quarter? **5¢**

If the value of the dimes is a multiple of the value of the quarters, there must be an even number of quarters, as a value ending in 5 cents cannot be evenly divisible by 10. Two quarters = 50 cents. 5 × 50 cents = 250 cents. To make 250 cents with dimes, 250 / 10 = 25 dimes needed. Therefore, 25 + 2 = 27 coins.

7. In how many different ways can 30 cents be made from any combination of quarters, dimes, nickels and/or pennies? **10¢**

There are 18 ways. Each possibility is listed in the format (number of quarters, number of dimes, number of nickels, number of pennies). They are (1, 0, 1, 0), (1, 0, 0, 5), (0, 3, 0, 0), (0, 2, 2, 0), (0, 2, 1, 5), (0, 2, 0, 10), (0, 1, 4, 0), (0, 1, 3, 5), (0, 1, 2, 10), (0, 1, 1, 15), (0, 1, 0, 20), (0, 0, 6, 0), (0, 0, 5, 5), (0, 0, 4, 10), (0, 0, 3, 15), (0, 0, 2, 20), (0, 0, 1, 25) and (0, 0, 0, 30).

8. Susan has exactly $1.50 in change. She has 30 total coins (pennies, nickels, dimes and/or quarters). Half of the coins are nickels. There are two-thirds as many pennies as there are nickels. How many dimes does Susan have? **10¢**

If half the coins are nickels, there are 30 / 2 = 15 nickels. If there are two-thirds as many pennies as there are nickels, there are 15 × (2/3) = 10 pennies. Set up an equation to represent the number of coins, $p + n + d + q = 30$, where p = pennies, n = nickels, d = dimes and q = quarters. Substituting what is known, $10 + 15 + d + q = 30$, which simplifies to $25 + d + q = 30$. Using the inverse operation, subtract 25 from both sides of the equation to get $d + q = 5$. Now, set up an equation to represent the value of the coins, $0.01p + 0.05n + 0.10d + 0.25q = 1.50$. Substituting what is known, $0.01(10) + 0.05(15) + 0.10d + 0.25q = 1.50$, which simplifies to $0.10 + 0.75 + 0.10d + 0.25q = 1.50$, then $0.85 + 0.10d + 0.25q = 1.50$. Using the inverse operation, subtract 0.85 from both sides of the equation to get $0.10d + 0.25q = 0.65$. These two equations make up a system of equations: $d + q = 5$ and $0.10d + 0.25q = 0.65$. Since the problem asks about the number of dimes, isolate the q in $d + q = 5$, in order to keep the variable d. Using the inverse operation, subtract d from both sides of the equation to get $q = 5 - d$. Substitute this into the other equation to get $0.10d + 0.25(5 - d) = 0.65$, which simplifies to $0.10d + 1.25 - 0.25d = 0.65$, then $1.25 - 0.15d = 0.65$ by combining like terms. Using the inverse operation, subtract 1.25 from both sides of the equation to get $-0.15d = -0.60$. Finally, using the inverse operation, divide by –0.15 on both sides of the equation to get $d = 4$. Therefore, Susan has 4 dimes.

9. A coin machine keeps 8.9 cents per $1 of coins inserted. This ratio also is maintained when only a fraction of a dollar is inserted. If Ashley inserts $50.50 worth of change into the ma chine, how much money will the machine give her back? **10¢**

The constant of proportionality between the ratio 0.089 : 1 and x : 50.50 is 50.50 / 1 = 50.50. So, 0.089 × 50.50 = 4.49 (rounded to the nearest hundredth), so $4.49 is the amount of money the machine will keep. So, Ashley will get back $50.50 – $4.49 = $46.01.

10. My piggy bank has only nickels, dimes and dollar bills. The ratio of nickels to dimes is 2:3, and the ratio of dimes to dollar bills is 10:1. What is the ratio of coins to dollar bills? Express your answer in the form *a:b*, where *a* and *b* are positive integers with no common factors greater than 1. **10¢**

In order to find the correct ratio, scale each provided ratio so that there are the same number of dimes in each. 10 × (2 : 3) = 20 : 30, and 3 × (10 : 1) = 30 : 3. So, when there are 30 dimes, there are 20 nickels, and 3 dollar bills, so 30 + 20 = 50 total coins to 3 dollar bills. This gives a ratio of 50 : 3.

11. There are equal numbers of pennies, nickels, dimes and quarters in a piggy bank. Two coins are pulled out, one at a time, and each coin is replaced before the next is drawn. What is the probability that the sum of the values of the two coins will be less than 15 cents? Express your answer as a common fraction. **10¢**

The probability should be written as (possible situations where the sum of the coin values is less than 15 cents) / (all possible combinations of choosing 2 coins). As the coins are replaced after they're drawn, all possible combinations of choosing 2 coins are PP, PD, DP, PN, NP, PQ, QP, DN, ND, DQ, QD, DD, NN, NQ, QN and QQ, where P = penny, D = dime, N = nickel and Q = quarter. Of these 16 possibilities, PP, PD, DP, PN, NP and NN are the combinations that would result in the sum of the values of the coins being less than 15 cents. This is 6/16 possibilities, which simplifies to 3/8.

12. John's piggy bank had $1.20 when he went to bed on Monday. On Tuesday morning, he put three coins in the bank. He put in three more coins in the afternoon and then three more in the evening. That night, there was a total of $2.20 in his bank. If no coin is worth more than 25 cents, what is the greatest amount, in cents, John could have put in the bank on Tuesday evening? **15¢**

In the evening, John added 3 coins to the piggy bank. To find the greatest amount he could have added with 3 coins, start with the coin of the highest value (the quarter) at his disposal. Assume John added 3 quarters, which is 75 cents. This would mean that with an additional 6 coins over the course of the day, John would have had to have added 25 cents (as he added $2.20 – $1.20 = $1 in total throughout the day). There is no combination of 6 coins that makes 25 cents, so this is not possible. Next, try 2 quarters, which is 50 cents. For the third coin, add a dime, since that is the coin with the next highest value, which would give a total of 60 cents. This would mean that with an additional 6 coins over the course of the day, John would have had to have added 40 cents. This is possible (with 2 dimes and 4 nickels). So, the greatest amount is 60 cents.

13. Jennifer, Mike and Carol each have a bunch of quarters. Jennifer and Mike have 26 quarters together. Jennifer and Carol have 20 quarters together. Mike and Carol have 22 quarters together. How many cents does Mike have? **15¢**

Convert the quarters into cents. 26 × 25 = 650, 20 × 25 = 500 and 22 × 25 = 550. Represent the scenario with equations. J + M = 650, where J = the number of cents Jennifer has and M = the number of cents Mike has. J + C = 500, where C = the number of cents Carol has. Finally, M + C = 550. Choose two equations with which to work that share one variable. For example, take M + C = 550 and J + C = 500. Subtract one equation from the oth-

er, (M + C = 550) – (J + C = 500) = M – J = 50. Using the inverse operation, add J to both sides of the resulting equation to get M = 50 + J. Substitute for M into the third equation, J + M = 650, to get J + (50 + J) = 650. Combine like terms to get 2J + 50 = 650. Using the inverse operations, subtract 50 from both sides of the equation and then divide each side of the equation by 2 to get J = 300. Finally, knowing J + M = 650, 300 + M = 650. Subtracting 300 from both sides gives that Mike has 350 cents.