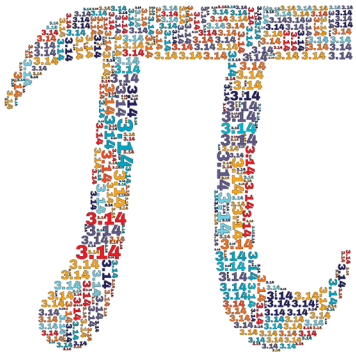
THE NATIONAL MATHCLUB

Pi Day



1. When π is written out in decimal form, what percent of the first 10 digits to the right of the decimal point are 5s?

2. Math people often use the fraction 22/7 to represent the value of π. If we had to use the common fraction $x/22$ to represent π, what integer-value of x would give us the closest approximation?

3. How many positive integers are less than 10π?

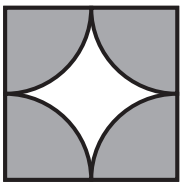
4. If Tony memorized 85 digits of π, and he recited them in exactly 18 seconds, then he recited them at the average rate of how many digits per second? Express your answer to the nearest whole number.

5. A particular circle's circumference is 4π inches, and its area is 4π square inches. What is the radius of the circle, in inches?

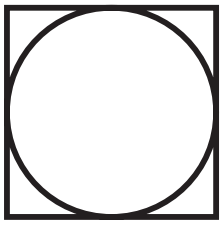
6. Pam Piper prepared a plum pie in a circular pan with a diameter of 9 inches. If the pie piece that Polly Peters purchased from Pam Piper was π square inches on the bottom, what percent of Pam Piper's prepared plum pie did Polly Peters purchase? Express your answer to the nearest whole number.

7. What is the area, in square units, of the circle that is centered at the origin and is tangent to the line $y = 7$? Express your answer in terms of π.

8. Four congruent quarter-circles are drawn inside a square of side length 4 cm, as shown. What is the area of the shaded portion of the square region, in sq cm? Express your answer in terms of π.



9. This square has an area of 49 square centimeters. What is the area of the inscribed circle, in sq cm? Express your answer as a decimal to the nearest tenth.



10. The formula for the total surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$, where r is the radius and h is the height. A particular solid right cylinder of radius 2 feet has a total surface area of 12π square feet. What is the height of the cylinder, in feet?

Club Leader Solution Guide!

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Pi Day

1. When π is written out in decimal form, what percent of the first 10 digits to the right of the decimal point are 5s?

   π = 3.1415926535..., as written to 10 decimal places. To the right of the decimal point, there are three 5s out of 10 digits, so 3/10 = 0.30 and 0.30 × 100 = 30%.

2. Math people often use the fraction 22/7 to represent the value of π. If we had to use the common fraction $x/22$ to represent π, what integer-value of x would give us the closest approximation?

   The constant of proportionality between these two ratios is π. So, $x = 22 \times \pi$, which is approximately 69.

3. How many positive integers are less than 10π?

   10π is approximately equal to 31.42. As 0 is not a positive integer, numbers 1 through 31 are the positive integers less than 10π, which is a total of 31 integers.

4. If Tony memorized 85 digits of π, and he recited them in exactly 18 seconds, then he recited them at the average rate of how many digits per second? Express your answer to the nearest whole number.

   The average rate of digits recited per second is 85 / 18, which is approximately 4.72 digits per second. This can be rounded to 5 digits per second.

5. A particular circle's circumference is 4π inches, and its area is 4π square inches. What is the radius of the circle, in inches?

   A circle's circumference is expressed by $C = \pi d$, where d = diameter. A circle's area is expressed by $A = \pi r^2$, where r = radius. Because both the circumference and area are equal to 4π in this scenario, $4\pi = \pi d = \pi r^2$. So, d = 4 inches, and therefore, r = 2 inches.

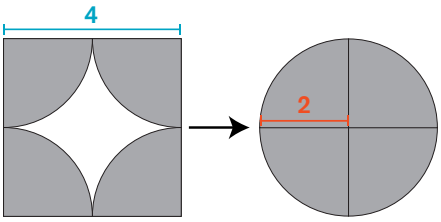
6. Pam Piper prepared a plum pie in a circular pan with a diameter of 9 inches. If the pie piece that Polly Peters purchased from Pam Piper was π square inches on the bottom, what percent of Pam Piper's prepared plum pie did Polly Peters purchase? Express your answer to the nearest whole number.

   With a 9-inch diameter, Pam Piper's entire plum pie has area $\pi r^2 = \pi(9/2)^2 = \pi(4.5)^2 = 20.25\pi$ square inches. If Polly Peters purchased a piece of size π square inches, the percentage of the whole pie represented is π / 20.25π = 1 / 20.25, which is equal to approximately 0.049, which, multiplied by 100, is 4.9%. Rounded to the nearest whole number, this is 5%.

7. What is the area, in square units, of the circle that is centered at the origin and is tangent to the line $y = 7$? Express your answer in terms of π.

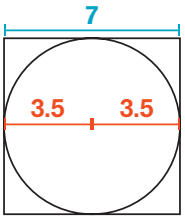
A circle that is centered at the origin and is tangent to the line $y = 7$ has a radius of 7 units. So, the area of this circle is equal to $\pi r^2 = \pi(7)^2 = 49\pi$ square units.

8. Four congruent quarter-circles are drawn inside a square of side length 4 cm, as shown. What is the area of the shaded portion of the square region, in sq cm? Express your answer in terms of π.



Four congruent quarter-circles would comprise one complete circle, so treat this question as if you were asked to find the area of the shaded circle. The side of the square is of length 4 cm. The side of the square is also equal to two radii of the circle, so 4 / 2 = 2 cm per radius. The area of the circle is represented by $\pi r^2 = \pi(2)^2 = 4\pi$ sq cm.

9. This square has an area of 49 square centimeters. What is the area of the inscribed circle, in sq cm? Express your answer as a decimal to the nearest tenth.



With an area of 49 square centimeters, the side length of the square is $\sqrt{49} = 7$ cm. Because the circle is inscribed, 7 cm is also the diameter of the circle, giving a radius of 3.5 cm. The area of the circle is represented by $\pi r^2 = \pi(3.5)^2 = 12.25\pi$, which is equal to approximately 38.5 sq cm.

10. The formula for the total surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$, where r is the radius and h is the height. A particular solid right cylinder of radius 2 feet has a total surface area of 12π square feet. What is the height of the cylinder, in feet?

Substituting the known values into the formula gives $12\pi = 2\pi(2)^2 + 2\pi(2)h$. This simplifies to $12\pi = 8\pi + 4\pi h$. Using the inverse operation, subtract 8π from both sides of the equation to get $4\pi = 4\pi h$. Again, using the inverse operation, divide by 4π on both sides of the equation to get $h = 1$ foot.