NEW!

# Patterns All Around

*Solving problems with visual and numerical patterns*

Seeing patterns and understanding how a particular pattern is being formed are great math skills to have! Some patterns are focused on numbers, and other patterns are visual. During this math club meeting, students can work together on the problem set provided. The problem set has fun problems that each involve a pattern that will lead to the answer.

## MATERIALS NEEDED

- Finish That Pattern! Game PDF presentation*
- Visual and Numerical Patterns Problem Set*
- Visual and Numerical Patterns Problem Set Solutions*
- Standard deck of cards
- Stopwatch

* Included in these club activity instructions

## BEFORE YOUR CLUB MEETING

We recommend that you read over the provided solutions before the club meeting so you are better able to steer the conversations of groups that may be having a tough time figuring out the patterns or how to use the patterns they have found.

## DURING YOUR CLUB MEETING

*Finish That Pattern! Game*

Divide your club members into small groups (2-4 people per group). Use the Finish That Pattern! PDF presentation, attached at the end of these instructions. Each slide contains the first few numbers of a pattern and a number of empty slots to be filled in. The group members must consult with each other, and then have one person raise her hand to answer. For each slide, the group that finishes the pattern correctly first will win a point. After going through as many slides as you would like (there are 20 slides total, but you can stop at any point, depending on your club meeting schedule), add up the total number of points for each group. The group with the most points is the winner!

*Visual Patterns Problems (#1-6)*

Keep your club members in small groups for the Visual and Numerical Patterns Problem Set. It may be a good idea to start with the visual patterns (#1-6), since many students find visual patterns easier to spot. It would be a good idea to guide groups that need more help with finding patterns. Have each group present its solution to one or two of the first six problems in the set.

*Numerical Patterns Problems (#7-10)*

Now that your club members have practiced identifying and understanding patterns in figures and tables, it's time to move on to numerical patterns! This may be trickier for some students, so you may need to provide more guidance along the way.

*Create That Pattern! Card Game*

Now you and your club members are ready for a fun patterns challenge in the form of the card game Create That Pattern! In this game, each club member will receive 10 cards and will be asked to create a numerical and/or visual pattern that encompasses as many of the 10 cards as possible. The great thing about this game is that it can be made easier or more difficult, depending on the ability level of your club members.

*Participants:* All club members (depending on how many club members you have, it may be best to have them work in pairs or small groups so that you do not run out of cards)

*Materials:* 1 or more standard decks of cards, depending on the number of participants

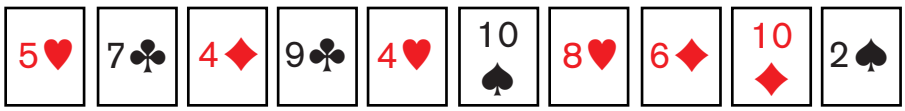
*Before you begin:*

1. Remove all face cards from the deck.
2. Aces are treated as 1s.
3. Determine how many rounds and how much time to spend for each round. This should depend on the ability level of your club members and on the time you have available.

*Setup:* Shuffle the deck of cards, and then give each player/pair/group 10 cards, all face down.

*How to play:*

1. When the Club Leader says "Begin," each player/pair/group should turn over their cards. If you'd like to give the students a time limit, you can use a timer or a clock and give them a few minutes.
2. In the time allowed, each player/pair/group must look at their 10 cards and create either a visual pattern or a numerical pattern that includes as many of the cards as possible.
3. Visual patterns (colors or suits) are worth 1 point per card used. (Decide whether just color can be a defining feature or whether all four suit shapes must be distinguished.) Numerical patterns are worth 2 points per card used. For example, consider the following set of 10 cards:




   A possible visual pattern is: ♠ ♥ ♦ ♠ ♦ ♥ ♣ ♥ ♦ ♣ and so on. This pattern **B**, **R**, **R** pattern is worth 10 points because all 10 cards are used. Some possible numerical patterns using the same cards may be:
   - 10, 9, 8, 7, 6, 5, 4 (subtracting 1 each time). Worth 14 points, because 7 cards are used.
   - 2, 4, 6, 8, 10 (adding 2 each time). Worth 10 points, because 5 cards are used.
   - 2, 4, 5, 7, 8, 10 (+2, +1, +2, +1, and so on). Worth 12 points, because 6 cards are used.
   - 10, 5, 8, 4, 7 (÷2, +3, ÷2, +3, and so on). Worth 10 points, because 5 cards are used.

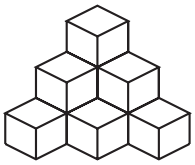
   ***NOTE: There must be at least two occurrences of the pattern. In other words, 10, 8, 7, 9, 4, 2 (−2, −1, +2, −5, −2) cannot be a pattern, because it does not repeat/occur twice.***
4. Have each player/pair/group explain their pattern(s) and determine the points for their best pattern of round 1.
5. Collect, reshuffle and redistribute the cards, and repeat steps 1-4 for the other rounds.
6. After completing all of the rounds your club wants to do, have each player/pair/group add up their points from each round. The player/pair/group with the most points after all rounds wins!
7. If there is a tie, the Club Leader will distribute 10 cards to each player/pair/group and give them only 15 seconds to come up with a pattern. The team that creates the pattern worth the most points in 15 seconds wins.

*For an easier game:*

- Require only visual patterns (using colors and/or suits).
- Deal 5 cards, instead of 10.
- Require only a 1-step numerical pattern.

# Visual and Numerical Patterns Problem Set

1. _____ The structure shows cubes stacked to form three layers. The pattern is continued to create a solid structure with the rows in each successive layer extending out one cube past the layer above them. How many cubes are needed to create a 10-layer structure?



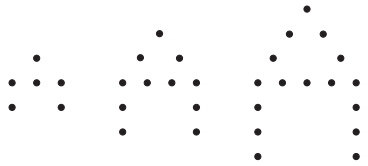


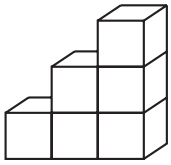
Figure 1 Figure 2 Figure 3

2. _____ If the dot pattern shown here is continued, how many dots will there be in Figure 4?

3. _____ This V-Dot pattern progresses as shown. As the pattern continues, how many dots will it take to make V-Dot 40?

V-Dot 1 V-Dot 2 V-Dot 3

4. _____ It takes 6 cubes to build a 3-level staircase. How many cubes are needed to build an 11-level staircase?



| | | | |
|---|---|---|---|
| 1 | | 3 | |
| | 2 | | |
| | P | | |
| | | | 4 |

5. _____ This sudoku-like figure is a 4-by-4 grid to be filled so that each of the digits 1, 2, 3 and 4 appears in each row and in each column. The 4-by-4 grid is divided into four 2-by-2 squares. Each of these 2-by-2 squares also is to contain each of the digits 1, 2, 3 and 4. What digit replaces P?

6. _____ If the positive integers are written in these five columns in the pattern shown, in which column will 89 be written?

| A | B | C | D | E |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | |
| | 8 | 7 | 6 | 5 |
| 9 | 10 | 11 | 12 | |
| | 16 | 15 | 14 | 13 |
| 17 | 18 | 19 | 20 | |
| | 24 | 23 | 22 | 21 |

7. _____ What is the value of the sum 1 + 4 + 7 + 10 + … + 91 + 94?

8. _____ The sequence 1, 3, 4, 7, … is continued by adding the two preceding terms to get the next term. What is the sum of the first 10 terms of this sequence?

9. _____ The first term in a sequence is 5, and each subsequent term in the sequence is the units digit of 2 more than the square of the preceding term. What is the 100th term in the sequence?

10. _____ Each term of a sequence, after the first term, is 3 less than the square of the preceding term. If the first term of the sequence is 2, what is the 2013th term?

# Visual and Numerical Patterns Problem Set SOLUTIONS

1. The first three layers have 1, 1 + 2, 1 + 2 + 3 cubes. The next layer will have 1 + 2 + 3 + 4 and so on. To get all 10 layers, the last layer will have 1 + 2 + ... + 10 cubes. The total will be 1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55 = **220** cubes. (*Did you do the Pascal's Triangle meeting earlier in this *Club Activity Book*? Notice these are the triangular numbers/3rd diagonal of Pascal's Triangle.)

2. The number of dots from figure to figure increases by 5 each time. A dot is added to each of the two diagonal roof segments, each of the two vertical wall segments and the horizontal roof segment. The numbers of dots making each of the first four figures are 6, 11, 16 and **21**.

3. Consider V-Dot 3. It has 4 "highest" points, 3 "lowest" points and 6 diagonal segments with 3 points each (that are not highest or lowest points). It seems (1) the number of lowest points is equal to the V-Dot number; (2) the number of highest points is equal to one more than the V-Dot number; (3) the number of diagonal segments is double the V-Dot number; and (4) the number of dots on each of those diagonal segments (not including the highest or lowest points) is equal to the V-Dot number. Notice each of these 4 assumptions holds for V-Dot 1, V-Dot 2 and V-Dot 3. Thus, V-Dot 40 will have 40 + 41 + 80(40) = **3281** total dots. Note: There are many other ways to describe the pattern.

4. The number of cubes added for each level is equal to the level number. So an 11-level staircase will have 1 + 2 + 3 + ... + 10 + 11 cubes. Notice you can pair up the numbers to form 5 pairs of 12, with a 6 left over. This is (5 × 12) + 6 = **66** cubes.

5. The top row needs a 2 and 4. Since the second column already has a 2, the top row must be 1, 4, 3, 2. This means the second row must be 3, 2, 4, 1, since only a 3 is needed in the upper-left quadrant, and there already is a 4 in the last column. This leaves only one blank in the last column, and the last column is 2, 1, 3, 4. Now, the second column must be 4, 2, 1, 3 or 4, 2, 3, 1. Since we already have a 3 in the third row (last column), P must be **1**.

| | | | |
|---|---|---|---|
| 1 | 4 | 3 | 2 |
| 3 | 2 | 4 | 1 |
| 4 | 1 | 2 | 3 |
| 2 | 3 | 1 | 4 |

6. **Column A** has all of the numbers that are 1 more than a multiple of 8, so it will contain 89.

7. Notice the numbers increase by 3 each time, so this is an arithmetic sequence. Similar to how we added the string of numbers in #4, we can pair numbers into sums of 95. Now, how many pairs do we have, and is there a number in the middle without a pairing? The numbers can be written as (3 × 0) + 1, (3 × 1) + 1, (3 × 2) + 1, (3 × 3) + 1, ..., (3 × 31) + 1. That means there are 32 terms, or 16 pairs, with no middle term. The sum is then 16 × 95 = **1520**.

8. According to the rule, the sequence becomes 1, 3, 4, 7, 11, 18, 29, 47, 76, 123. The sum is **319**.

9. According to the rule, the sequence becomes 5, 7 (from $5^2 + 2 = \cancel{2}7$), 1 (from $7^2 + 2 = \cancel{5}1$), 3 (from $1^2 + 2 = 3$), 1 (from $3^2 + 2 = \cancel{1}1$), 3 (from $1^2 + 2 = 3$), 1, 3, 1, ... . The even terms (after the 2nd) are each 3, so the 100th term is **3**.

10. According to the rule, the sequence becomes 2, 1 (from $2^2 - 3 = 1$), −2 (from $1^2 - 3 = -2$), 1 (from $(-2)^2 - 3 = 1$), −2, 1, −2 and so on. The odd terms (after the 1st) are each −2, so the 2013th term is **−2**.

# Patterns All Around

## Finish That Pattern!

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# Finish That Pattern!

**Here's how to play:**

- Each slide contains the first few numbers of a pattern and some empty slots to be filled in, based on that pattern.

  *For example: 2, 4, 6, 8 ___, ___, ___, ___.*
- Figure out the pattern that is in the number sequence, then determine the numbers that should be in the missing slots.

  *Using the same example: pattern is +2 → 2, 4, 6, 8, 10, 12, 14, 16*
- The first group/individual to correctly state the pattern's missing numbers in the correct order wins 1 point.
- There's a "sample answer" provided for each pattern, but you may come up with a different answer that also works. As long as you can explain your pattern and it repeats at least twice, it can be accepted.
- At the end, the group/individual with the most points wins!

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# Finish That Pattern!

## Pattern #1:

**6, 9, 7, 10, 8, 11, __, __, __.**

# Finish That Pattern!

## Sample Answer:

**6, 9, 7, 10, 8, 11, <u>9</u>, <u>12</u>, <u>10</u>.**

***(pattern: +3, -2…)***

# Finish That Pattern!

## Pattern #2:

7, 14, 28, 56, __, __, __, __.

# Sample Answer:

**7, 14, 28, 56, <u>112</u>, <u>224</u>, <u>448</u>, <u>896</u>.**

*(pattern: x2…)*

# Finish That Pattern!

## Pattern #3:

**96, 48, 50, 25, 27, __, __.**

# Sample Answer:

**96, 48, 50, 25, 27, <u>13.5</u>, <u>15.5</u>.**

***(pattern: ÷2, +2…)***

# Finish That Pattern!

## Pattern #4:

1, 2, 4, 8, 16, __, __, __.

# Sample Answer:

**1, 2, 4, 8, 16, 32, 64, 128.**

*(pattern: powers of 2 ($2^0$, $2^1$, $2^2$…)*

# Finish That Pattern!

## Pattern #5:

17, 35, 71, 143, __, __, __.

# Sample Answer:

**17, 35, 71, 143, <u>287</u>, <u>575</u>, <u>1151</u>.**

***(pattern: x2 +1…)***

# Pattern #6:

2, 4, -4, 16, 8, 64, __, __, __.

# Finish That Pattern!

## Sample Answer:

**2, 4, -4, 16, 8, 64, <u>56</u>, <u>3136</u>, <u>3128</u>.**

*(pattern: #$^2$, -8…)*

# Pattern #7:

3, 6, 1, 4, -1, __, __, __.

# Finish That Pattern!

## Sample Answer:

**3, 6, 1, 4, -1, <u>2</u>, <u>-3</u>, <u>0</u>.**

***(pattern: +3, -5…)***

# Finish That Pattern!

## Pattern #8:

6075, 2025, 675, __, __, __.

# Finish That Pattern!

## Sample Answer:

**6075, 2025, 675, <u>225</u>, <u>75</u>, <u>25</u>.**

***(pattern: ÷3…)***

# Finish That Pattern!

## Pattern #9:

**99, 88, 78, 69, 61, __, __, __.**

# Sample Answer:

**99, 88, 78, 69, 61, <u>54</u>, <u>48</u>, <u>43</u>.**

***(pattern: -11, -10, -9…)***

# Pattern #10:

90, 60, 30, __, __, __, __.

# Finish That Pattern!

## Sample Answer:

**90, 60, 30, <u>0</u>, <u>-30</u>, <u>-60</u>, <u>-90</u>.**

***(pattern: -30…)***

# Pattern #11:

1, 16, 8, 23, 11.5, __, __, __.

# Sample Answer:

**1, 16, 8, 23, 11.5, <u>26.5</u>, <u>13.25</u>, <u>28.25</u>.**

***(pattern: +15, ÷2…)***

# Pattern #12:

40, 40.1, 39, 39.1, __, __, __.

# Sample Answer:

**40, 40.1, 39, 39.1, <u>38</u>, <u>38.1</u>, <u>37</u>.**

***(pattern: +0.1, -1.1…)***

# Pattern #13:

$\frac{9}{10}$ , $\frac{8}{13}$ , $\frac{7}{16}$ , __, __, __.

# Finish That Pattern!

## Sample Answer:

$9/10$ , $8/13$ , $7/16$ , $\underline{6/19}$ , $\underline{5/22}$ , $\underline{4/25}$.

*(pattern: decrease numerator by 1 and increase denominator by 3…)*

# Pattern #14:

800, 200, 50, __, __, __.

# Finish That Pattern!

## Sample Answer:

**800, 200, 50, <u>12.5</u>, <u>3.125</u>, 0<u>.78125</u>.**

***(pattern: ÷4…)***

# Finish That Pattern!

## Pattern #15:

90, 90.8, 91.5, 92.1, __, __, __.

# Finish That Pattern!

## Sample Answer:

**90, 90.8, 91.5, 92.1, <u>92.6</u>, <u>93</u>, <u>93.3</u>.**

***(pattern: +0.8, +0.7, +0.6…)***

# Pattern #16:

1, 3, 6, 10, 15, __, __, __, __.

# Finish That Pattern!

## Sample Answer:

**1, 3, 6, 10, 15, <u>21</u>, <u>28</u>, <u>36</u>, <u>45</u>.**

***(pattern: +1, +2, +3…)***

# Finish That Pattern!

## Pattern #17:

2000, 1000, 1500, __, __, __.

# Sample Answer:

**2000, 1000, 1500, <u>750</u>, <u>1125</u>, <u>562.5</u>.**

***(pattern: ÷2, x1.5…)***

# Finish That Pattern!

## Pattern #18:

100, 109, 119, 130, 142, __, __.

# Finish That Pattern!

## Sample Answer:

**100, 109, 119, 130, 142, <u>155</u>, <u>169</u>.**

***(pattern: +9, +10, +11…)***

# Pattern #19:

8; 800; 80,000; __; __.

# Sample Answer:

**8; 800; 80,000; <u>8,000,000</u>; <u>800,000,000</u>.**

***(pattern: x100…)***

# Pattern #20:

4, 7, 0, 3, -4, -1, __, __, __.

# Finish That Pattern!

## Sample Answer:

**4, 7, 0, 3, -4, -1, <u>-8</u>, <u>-5</u>, <u>-12</u>.**

***(pattern: +3, -7…)***