

# Pascal's Triangle

accompanies the Parts 1 & 2 Handout  
and the Part 3 Handout

THE NATIONAL  
**MATHCLUB**



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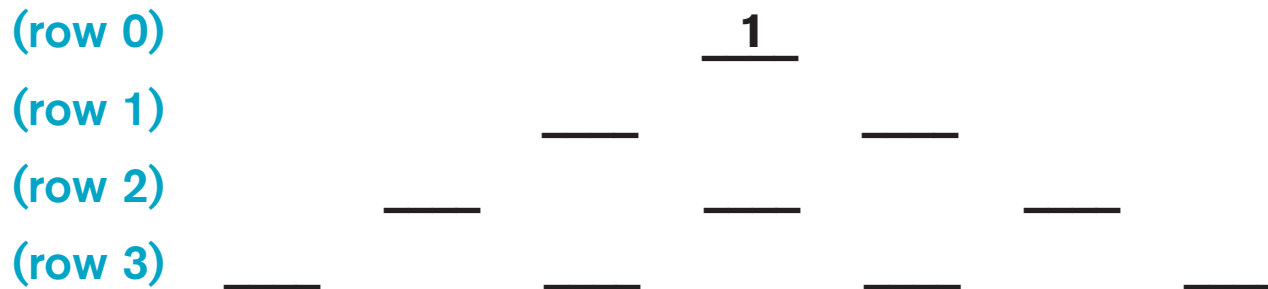
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1. The first row has one entry... and the entry is 1.

Note: Though this is the first row, it is referred to as **row 0**.



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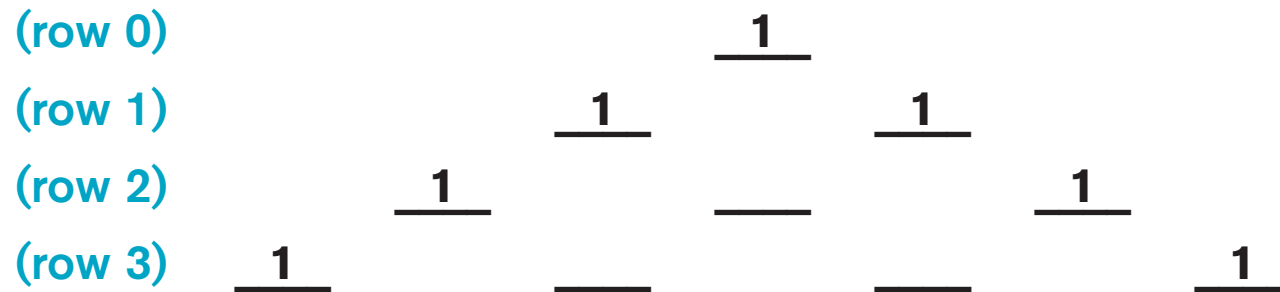
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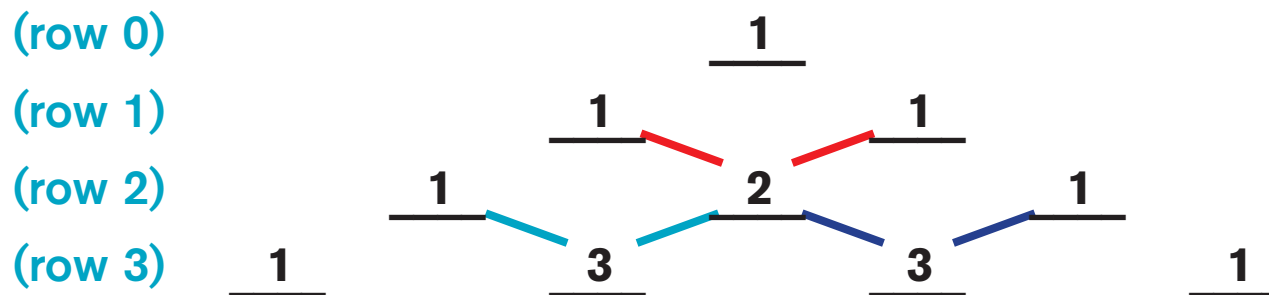
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2. Every row after row 0 has a first entry of 1 and a last entry of 1.

3. For every row after row 1, the first entry is 1; the last entry is 1; all other entries are the sum of the number diagonally to the right and the number diagonally to the left in the row above it.



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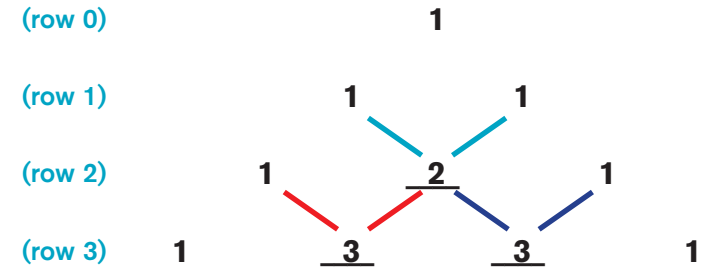


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**PART 1: PASCAL'S TRIANGLE HANDOUT.** Use what you've learned about Pascal's Triangle to fill in row 2 through row 10. Ignore the ovals and rectangles for Part 1.

(row 0) 1

(row 1) 1 1

(row 2) — —

(row 3) — — —

(row 4) — — — —

(row 5) — — — — —

(row 6) — — — — — —

(row 7) — — — — — — —

(row 8) — — — — — — — —

(row 9) — — — — — — — — —

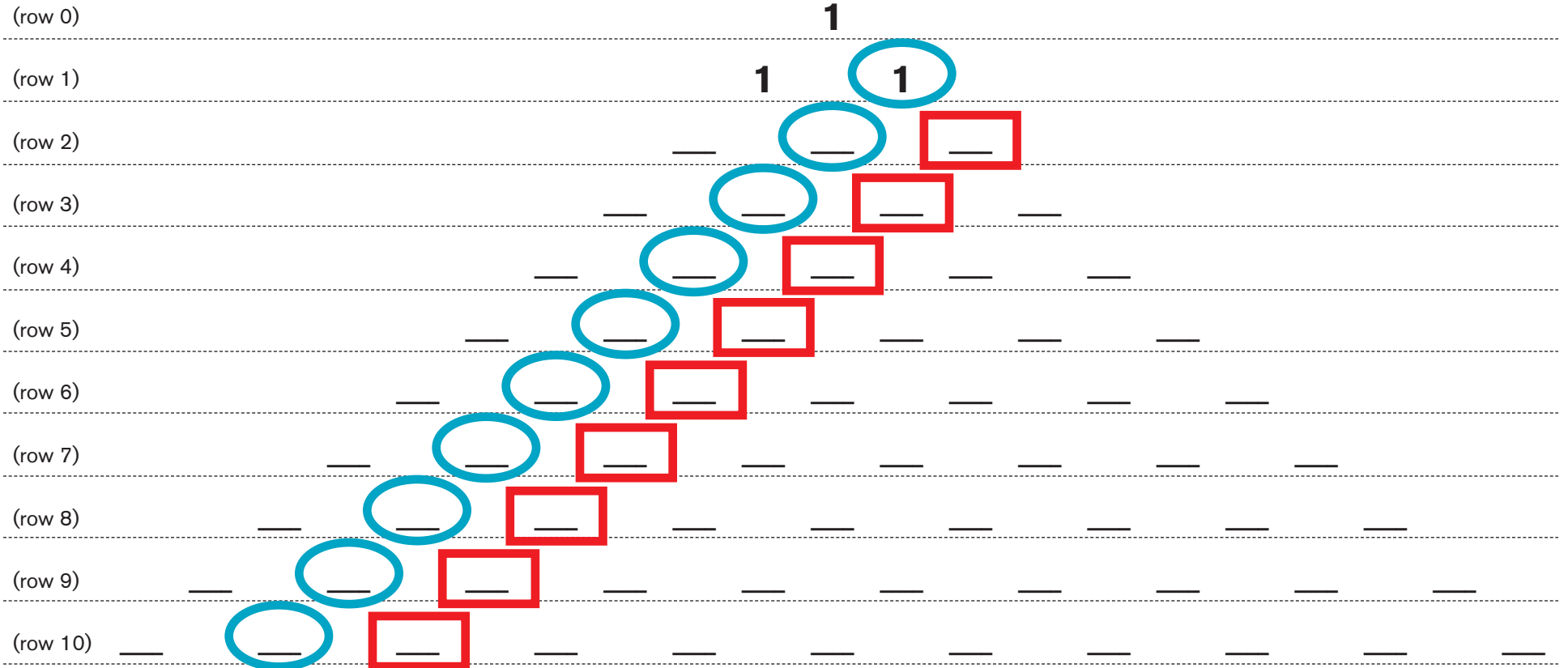
(row 10) — — — — — — — — — —



# Pascal's Triangle

## Parts 1 & 2 Handout

**PART 1: PASCAL'S TRIANGLE BASICS.** Use what you've learned about Pascal's Triangle to fill in row 2 through row 10. Ignore the ovals and rectangles for Part 1. Remember, (1) the first and last numbers in each row are 1 and (2) every other entry is the sum of the two numbers above it (diagonally right and diagonally left).

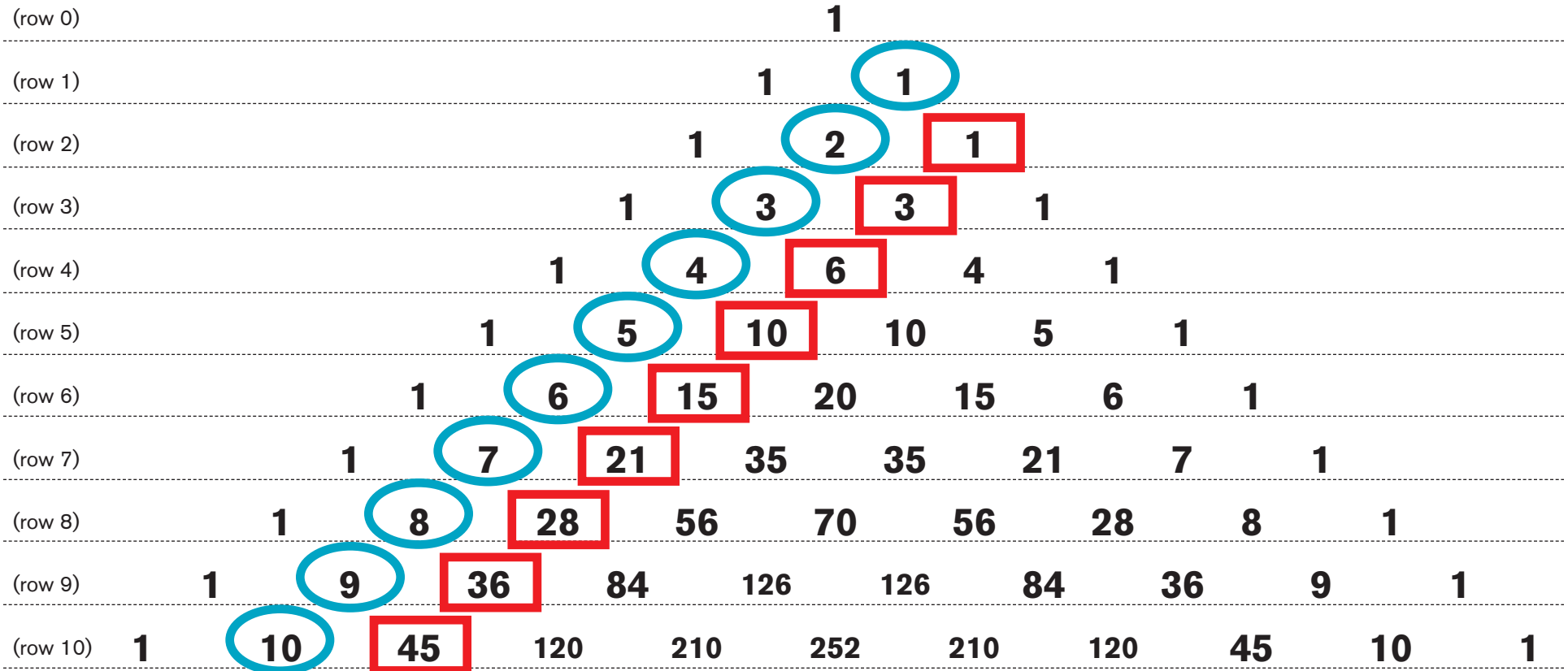






# Pascal's Triangle **Answer Key**

Parts 1 & 2 Handout



Now that you have completed Part 1....

**PART 2: IDENTIFYING PATTERNS IN PASCAL'S TRIANGLE.** Use what you've done in Part 1 to answer the questions below.

1. What pattern(s) do you notice in the numbers of each row?
2. Add the numbers of each row. What do you notice about these sums?
3. What do you notice about the numbers in the second diagonal (ovals)?
4. What do you notice about the numbers in the third diagonal (rectangles)?



# Pascal's Triangle **Answer Key**

Parts 1 & 2 Handout

(row 0)					1										
(row 1)					1	1									
(row 2)					1	2	1								
(row 3)					1	3	3	1							
(row 4)					1	4	6	4	1						
(row 5)					1	5	10	10	5	1					
(row 6)					1	6	15	20	15	6	1				
(row 7)					1	7	21	35	35	21	7	1			
(row 8)					1	8	28	56	70	56	28	8	1		
(row 9)					1	9	36	84	126	126	84	36	9	1	
(row 10)					1	10	45	120	210	252	210	120	45	10	1

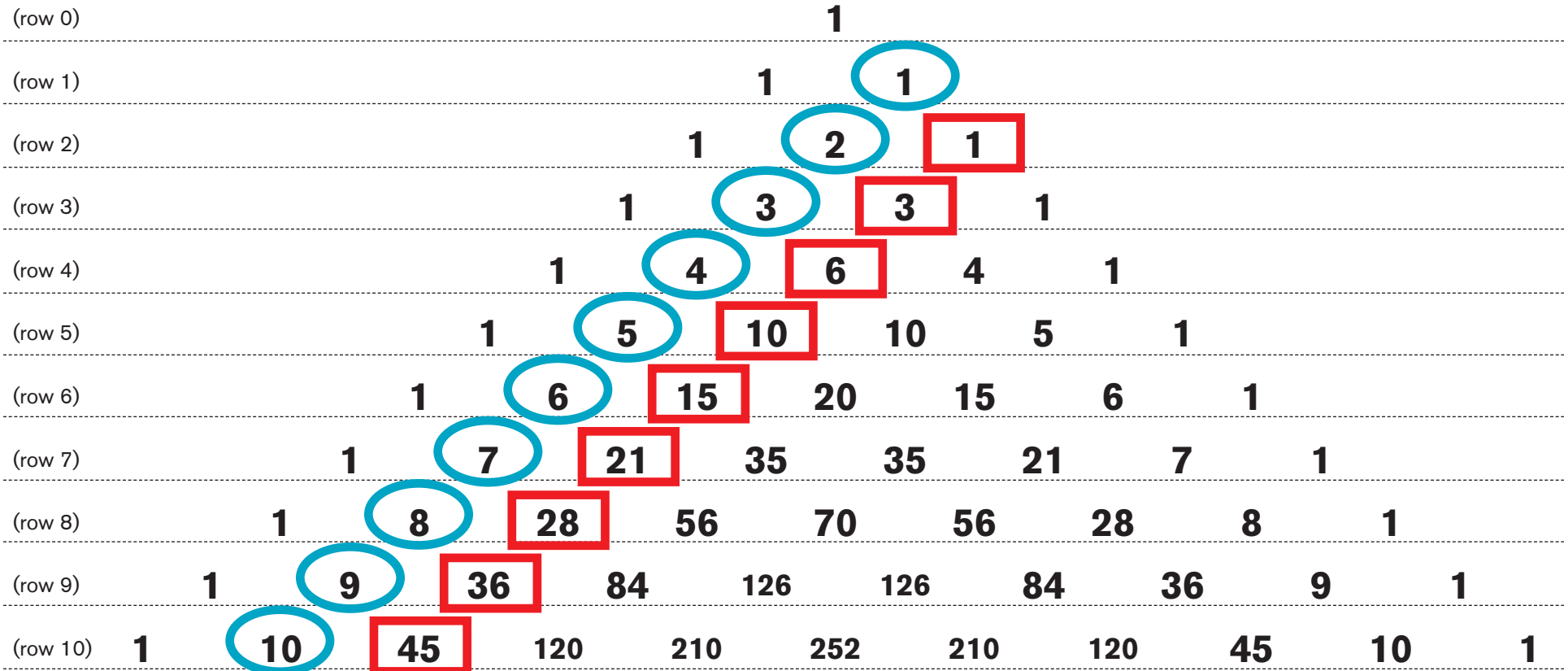
**PART 2: IDENTIFYING PATTERNS IN PASCAL'S TRIANGLE.** Use what you've done in Part 1 to answer the questions below.

1. What pattern(s) do you notice in the numbers of each row? **The first and last numbers are the same; the second and second-to-last numbers are the same; the third and third-to-last numbers are the same; and so on.** There are other patterns students may find... there is a "center" number in all of the even rows (row 0, row 2, row 4, etc.) and there are duplicate numbers in the two center-most spots in the odd rows (row 1, row 3, row 5, etc.). The numbers in each row increase until they get to the middle of the row and then they decrease. The number of entries in each row is one more than the row number, so, for example, row 6 has  $6 + 1 = 7$  entries.



# Pascal's Triangle Answer Key

Parts 1 & 2 Handout



**PART 2: IDENTIFYING PATTERNS IN PASCAL'S TRIANGLE.** Use what you've done in Part 1 to answer the questions below.

1. What pattern(s) do you notice in the numbers of each row?
2. Add the numbers of each row. What do you notice about these sums? **The sums are 1, 2, 4, 8, 16, 32, 64, 128, 256, 512 and 1024. The sum of each row is double the sum of the row above it; and the sums are all powers of 2. The sum of the numbers in row  $n$  is  $2^n$ . For example, the sum of the numbers in row 10 is  $2^{10} = 1024$ .**



# Pascal's Triangle Answer Key

Parts 1 & 2 Handout

(row 0)											1										
(row 1)											1	1									
(row 2)											1	2	1								
(row 3)											1	3	3	1							
(row 4)											1	4	6	4	1						
(row 5)											1	5	10	10	5	1					
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(row 10)											1	10	45	120	210	252	210	120	45	10	1

**PART 2: IDENTIFYING PATTERNS IN PASCAL'S TRIANGLE.** Use what you've done in Part 1 to answer the questions below.

1. What pattern(s) do you notice in the numbers of each row?.
2. Add the numbers of each row. What do you notice about these sums?
3. What do you notice about the numbers in the second diagonal (ovals)? **They are the positive integers and correspond to their row numbers... row 1 has a 1 in the second diagonal, row 2 has a 2, and so on. We can determine that row 14, for example, starts with 1 and 14 and ends with 14 and 1.**



# Pascal's Triangle **Answer Key**

Parts 1 & 2 Handout

(row 0)					1										
(row 1)					1	1									
(row 2)					1	2	1								
(row 3)					1	3	3	1							
(row 4)					1	4	6	4	1						
(row 5)					1	5	10	10	5	1					
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1. What pattern(s) do you notice in the numbers of each row?
2. Add the numbers of each row. What do you notice about these sums?
3. What do you notice about the numbers in the second diagonal (ovals)?
4. What do you notice about the numbers in the third diagonal (rectangles)? **Starting with 1 in the third diagonal, the pattern is: add 2 (to get 3); add 3 (to get 6); add 4 (to get 10); add 5 (to get 15); and so on. These are the triangular numbers.**



# Other Great Patterns in Pascal's Triangle

(row 0)				1									
(row 1)			1		1								
(row 2)			1		2		1						
(row 3)			1		3		3		1				
(row 4)			1		4		6		4		1		
(row 5)			1		5		10		10		5		1

## COMBINATIONS

Consider row 4: 1, 4, 6, 4, 1

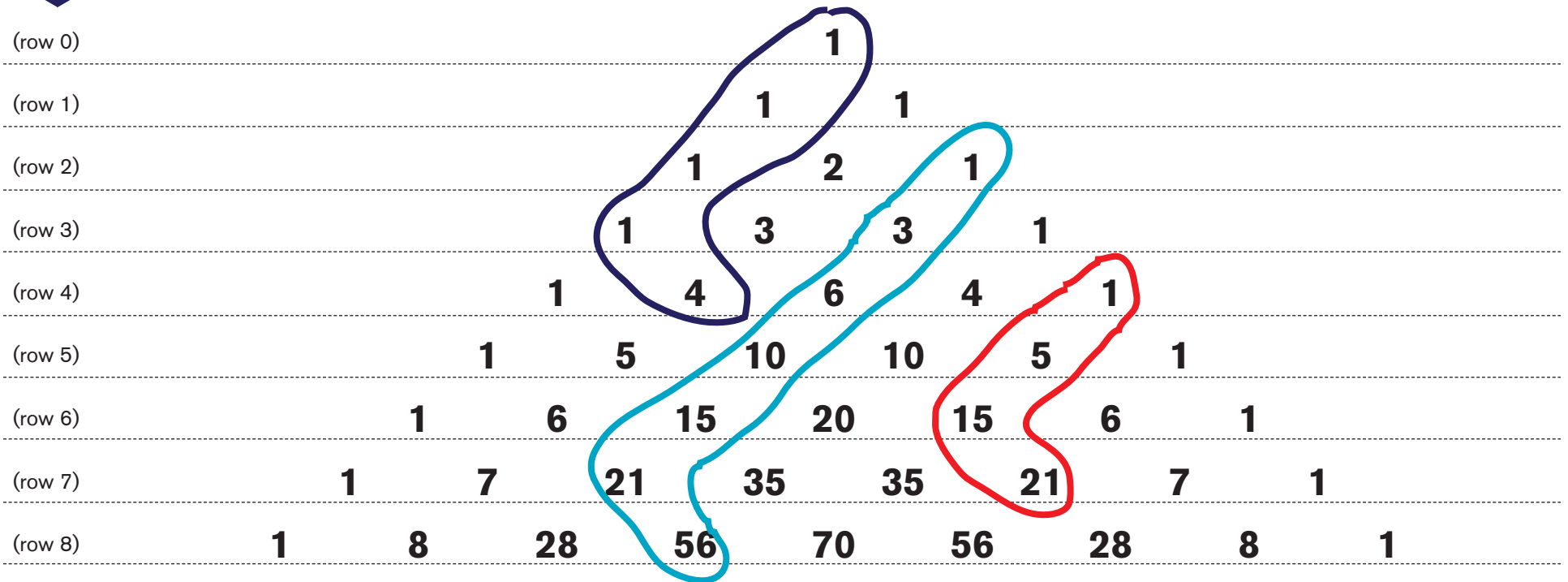
Notice these entries are equal to  ${}_4C_0$  (or "4 choose 0"),  ${}_4C_1$ ,  ${}_4C_2$ ,  ${}_4C_3$ ,  ${}_4C_4$ .

When trying to determine  ${}_5C_3$ , one could form Pascal's Triangle, go to row 5 and find the 3 + 1 = 4th entry... circled above.

Visually, you can see that  ${}_5C_3 = {}_5C_2$ , which makes sense... picking 3 items from 5 to "take" is the same as picking 2 items from 5 to "leave." Similarly,  ${}_5C_0 = {}_5C_5$  and  ${}_5C_1 = {}_5C_4$ .



# Other Great Patterns in Pascal's Triangle



## THE BOOT PATTERN

Start with any 1 on the outer right edge of the triangle. If you continue down the diagonal to the left, the sum of the numbers you “capture” on the diagonal will be equal to the number that is “in the toe” on the next row down and diagonally to the right. Notice this makes a boot shape.

$$1 + 1 + 1 + 1 = 4$$

$$1 + 3 + 6 + 10 + 15 + 21 = 56$$

$$1 + 5 + 15 = 21$$



# Other Great Patterns in Pascal's Triangle

(row 0)				1						$2^0 = 1$							
(row 1)				1		1				$2^1 = 2$							
(row 2)				1		2		1		$2^2 = 4$							
(row 3)				1		3		3		1	$2^3 = 8$						
(row 4)				1		4		6		4		1	$2^4 = 16$				
(row 5)				1		5		10		10		5		1	$2^5 = 32$		
(row 6)				1		6		15		20		15		6		1	$2^6 = 64$

## THE POWER OF 2 PATTERN

The sum of the entries in each row is a power of 2. The sum of row  $n$  is  $2^n$ .