Opposite Day (Jan. 25) Meeting (Inverse Operations)

Topic

This meeting's topic is inverse operations or "doing the opposite."

Materials Needed

- 50 pieces of paper (scraps or half-sheets will work)
- Markers
- Wall or board on which to tape Operation Directions
- Tape
- A copy of the problems provided throughout this lesson and/or online at www.mathcounts.org on the MCP Members Only page of the Club Program section – optional
- Prizes for students – optional

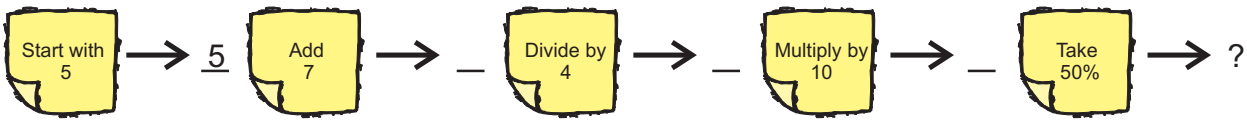
Meeting Plan

The meeting plan begins with students doing some mental math. Below are some "expression strings" that you can use first, but feel free to make up some of your own. Note that students should do the operations in the order that you are saying them — we are not worried about any other order of operations right now. We recommend doing this without a calculator, and giving students only enough time to keep up, but not to share their answers in the middle of an expression string. When you reach the end of an expression string, students then can share the answers they have. When there are a variety of answers, take this great teaching opportunity to go back as a group and figure out from where the discrepancies came. It's great to have some prize incentives for active participation throughout the activity.

Let students know that each calculation should produce either an integer or a fraction. (Students can convert values into decimals, but they will have to get them back into fraction form in the end if they do so.)

Each expression string has a Start Number (which will be called the Mystery Number later on), a series of Operation Directions and then a Final Number. Note: When the Operation Direction says "take x%," we intend for the students to calculate x% of the current number, which then becomes their new number. So if a student has 10 and is supposed to then "take 70%," the result is 7.

Ideally, each Operation Direction can be written on a piece of paper and taped to the board as you go. The papers are put up one at a time, with the next paper going up after you feel your students have had enough time to complete the current step. Leave enough room to then go back and fill in the step-by-step answers along the way. When doing this first example, we envision the chalkboard looking like this at the end:



Here is how this is read:

Start with 5, then add 7 (pause for them to get the answer), then divide by 4 (pause for

them to get the answer), then multiply by 10 (pause for them to get the answer), then take 50%. What is your final number? **Answer: 15**

Once students have had a few seconds to determine the answer, ask them for their outcomes and fill in the empty spaces in the string.

Here are a few more that we created, but feel free to make up your own and use operations that your students will feel comfortable with. If they have worked extensively with powers, roots, absolute value, etc., then those can be included, too.

Start with 2, then multiply by 25 (pause), then add 31 (pause), then take the square root (pause), then multiply by 2/3. What is your final number? **Answer: 6**

Start with 8, then add 4 (pause), then divide by 5 (pause), then multiply by 5/3. What is your final number? **Answer: 4**

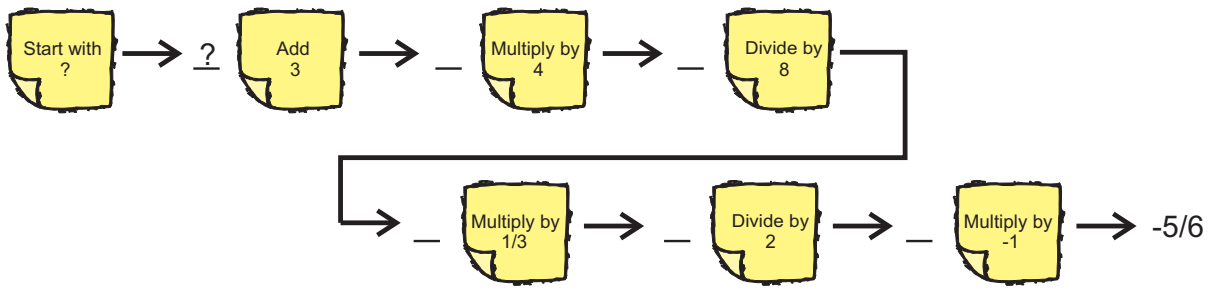
Start with 2/3, then subtract 6 (pause), then multiply by 1/4 (pause), then square the number. What is your final number? **Answer: 16/9**

Start with 50, then subtract 35 (pause), then multiply by 7 (pause), then divide by 21 (pause), then take 120% (pause), then divide by 2/3 (pause), then square that number (pause), then add 9 (pause), then take 20% (pause), then multiply by -2 (pause), then take the absolute value. What is your final number? **Answer: 36**

Let students come up with their own, too, to try to stump the club. You may wish to put some restrictions on their expression strings, such as having five or fewer steps.

Now for Opposite Day… Once students are familiar with expression strings and how they work, let them know that you are going to give them the same type of expression string, but this time you are going to give them the Final Number, and they have to go in the opposite direction to get the Start Number. This exercise focuses on the concept of inverse operations, which is sometimes referred to as “doing the opposite.”

With Operation Directions on pieces of paper, you can create an arrangement similar to the one seen here. We recommend working through at least one of these with your students before setting them off on their own. Rather than putting up each piece of paper one at a time as you go forward, the entire string should be put up at the same time with the answer written at the end.



Here is how the example above would be read:

We’re going to start with the Mystery number, then if we add 3, and then we multiply by 4,

and then we divide by 8, and then we multiply by 1/3, and then we divide by 2, and then we multiply by -1, our final answer is -5/6. What was the Mystery Number?

This can be intimidating, so do not hesitate to work through this with the students. You certainly can make it shorter if you feel that would be better. We made this one long to give the students a lot of practice before they try it on their own. Here is the solution:

Going in the opposite direction and performing the opposite operations, we have: Start with -5/6, then divide by -1 to get 5/6, then multiply by 2 to get 10/6 = 5/3, then multiply by 3 (or divide by 1/3) to get 5, then multiply by 8 to get 40, then divide by 4 to get 10, then subtract 3 to get 7. **The Mystery Number was 7.**

Here are two more you can give your students:

Start with the Mystery Number, then add 4, then divide by -2, then multiply by -1/3, then add 8, then subtract 2 and your final answer is 8. What was the Mystery Number? **Answer: 8**

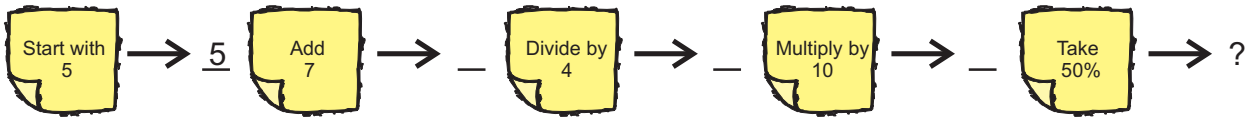
Start with the Mystery Number, then multiply by 8, then add 11, then multiply by 1/3, then add 3, then take 125%, then divide by 7 and your final answer is 10/7. What was the Mystery Number? **Answer: 1/2**

Possible Next Steps

Separate your club into two teams. Each team can attempt to come up with an expression string that will stump the other team. Each team should determine a Start Number (that they don't tell the other team), create up to 8 Operation Directions to change that Start Number and then determine the Final Number. Each team then will give their expression string and Final Number to the other team to see if it can get the correct Start/Mystery Number. (It is best if you check the expression strings before the teams swap so that any necessary edits can be done ahead of time.)

Notice that if you originally put "square the number" or "take the absolute value of a number" in the expression string going forward, then there could be multiple answers when the strings are worked in the opposite direction. For instance, if you "take the absolute value" and get 2, then when you go backwards, you can't be sure if you had a 2 or -2 before you took the absolute value. Similarly, if you "square the number" and get 9 going forward in the string, then when you go backwards, you can't be sure if you had a 3 or a -3 before you squared the number. You may or may not want to address this with your students, depending on their comfort level with those concepts.

Opposite Day Meeting
Problem Set - Coach Examples



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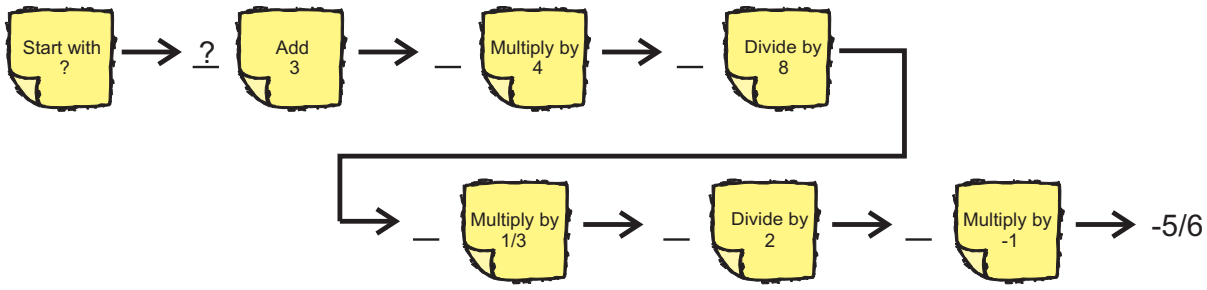

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