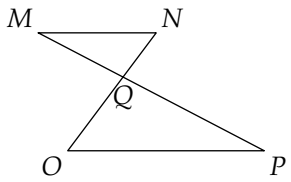
*Activity Sheet for the March, 2014, MATHCOUNTS Mini*

# Warm-Up!

Try these problems before watching the lesson.

1. In the diagram below, $\overline{MN} \parallel \overline{OP}$, $MN = 12$, and $OP = 20$. If $ON = 24$, then what is $QN$?

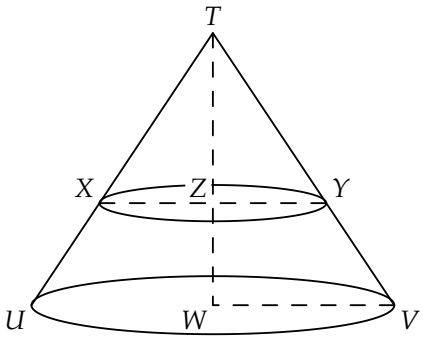



2. The area of $\triangle STU$ is 45. Points $P$ and $Q$ are on sides $\overline{ST}$ and $\overline{SU}$, respectively, such that $\overline{TU} \parallel \overline{PQ}$. If $SP = 2PT$, what is the area of $\triangle SPQ$?

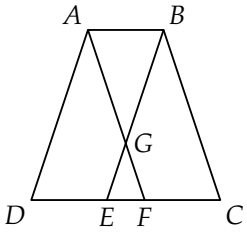
3. The height of a right circular cone is three times its radius. If the circumference of the base of the cone is $6\pi$, what is the volume of the cone?

# The Problems

**First Problem:** The two cones shown have parallel bases and common apex $T$. $TW = 32$ m, $WV = 8$ m, and $ZY = 5$ m. What is the volume of the frustum with circle $W$ and circle $Z$ as its bases?



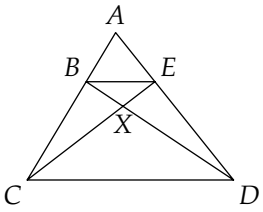

**Second Problem:** In isosceles trapezoid $ABCD$, shown here, $AB = 4$ units and $CD = 10$ units. Points $E$ and $F$ are on $\overline{CD}$ with $\overline{BE}$ parallel to $\overline{AD}$ and $\overline{AF}$ parallel to $\overline{BC}$. $\overline{AF}$ and $\overline{BE}$ intersect at point $G$. What is the ratio of the area of triangle $EFG$ to the area of trapezoid $ABCD$?



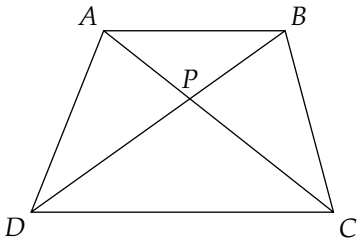
## Follow-up Problems

4. Triangle $PQR$ is a right triangle with $\angle Q = 90°$, $PQ = 3$, and $QR = 4$. Points $S$, $T$, and $U$ are on sides $\overline{PQ}$, $\overline{PR}$, and $\overline{QR}$, respectively, such that $QSTU$ is a square. Find the length of $\overline{ST}$. Express your answer as a common fraction.

5. In the diagram below, $\overline{BE} \parallel \overline{CD}$. If the area of trapezoid $BCDE$ is 8 times the area of $\triangle ABE$, and the area of $\triangle CDX$ is 27 square units, then what is the area of $\triangle ACD$?




6. Trapezoid $ABCD$, with $\overline{AB} \parallel \overline{CD}$, is shown below. Suppose the area of $\triangle PAB$ is $a^2$, and the area of $\triangle PCD$ is $b^2$, where $a$ and $b$ are positive. Explain why the area of $ABCD$ must be $(a+b)^2$.




7. Find a formula for the volume of a frustum of a right cone whose bases have radii $r$ and $s$, and whose height is $h$.

## Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).