MATHCOUNTS® Minis
www.mathcounts.org
presented by
Art of Problem Solving
www.artofproblemsolving.com

Activity Sheet for the May, 2019, MATHCOUNTS Mini

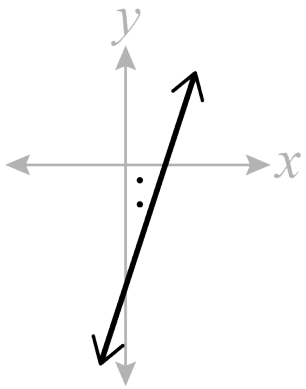
Warm-Up!

Try these problems before watching the lesson.

1. Parallelogram $ABCD$ has $A(0,1)$, $B(2,1)$ and $C(4,0)$ as three of its vertices. What is the sum of the coordinates of point D?
2. A rectangle has vertices $(0,0)$, $(7,0)$, $(7,4)$ and $(0,4)$. How many lattice points are in the interior of the rectangle? (A lattice point is a point with integer coordinates.)
3. What is the area, in square units, of the triangle bounded by $y = 0$, $y = x + 4$ and $x + 3y = 12$?
4. B and C are constants such that the graph of $x + By = C$ consists of all points that are equidistant from $(-2, 3)$ and $(6, -7)$. Find B.

The Problems

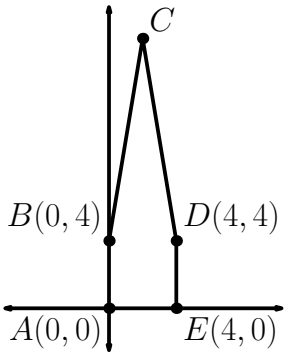
First Problem: The right triangle bounded by the x- and y-axes and the line $3x - y = 6$ contains 2 lattice points in its interior. How many lattice points will be contained in the interior of the triangle bounded by the x- and y-axes and the line $3x - y = 24$?



Second Problem: Two lines with slopes m and n, with $m > n > 0$, intersect at the origin. The line $y = x$ bisects the angle between the two lines. If $m + n = 2\sqrt{65}$, what is the value of $m - n$?

Follow-up Problems

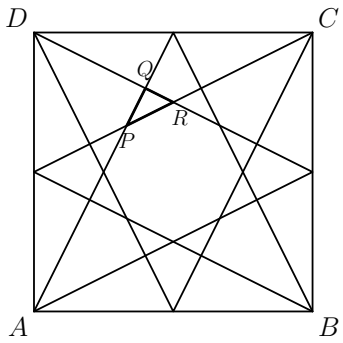
5. Pentagon $ABCDE$ has a vertical line of symmetry and has an area of 40 square units. How many lattice points are in the interior of the pentagon?



6. Triangle ABC has vertices $A(0, 0)$, $B(0, 3)$ and $C(5, 0)$. A point P inside the triangle is $\sqrt{10}$ units from point A and $\sqrt{13}$ units from point B. How many units is P from point C? Express your answer in simplest radical form.

7. Points $A(5, 3)$, $B(2, 0)$, $C(-2, 4)$ and $D(x, 2x)$ are in the plane such that point D is equidistant from the sides of $\angle ABC$. What is x?

8. In square $ABCD$, each vertex is connected to the midpoints of its two opposite sides, as shown. What is $\frac{QR}{PQ}$? Express your answer as a common fraction.



Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).