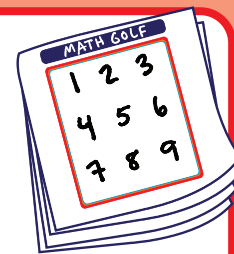
MATH GOLF

Everything You Need to Play

Materials



Math Golf Game Board

- 1 game board per student. Students can play solo (trying for a personal best score) or in groups of two or more. Find a copy of the game board below.
- Using a paper copy of the board is fine, but you may also want to make a laminated copy for multiple uses. A sheet protector is an easy substitute for lamination!

Standard Six-Sided Dice

- 2 six-sided dice per group of students.
- The MATHCOUNTS Club App is available in lieu of physical dice. If the app is used, one device is needed per group of students.

Writing Utensils and Scratch paper

- 1 writing utensil per student—pencils or dry erase markers (for laminated boards).

Rules

Math Golf is a game of dice rolling and arithmetic. It can be played with any number of people. Each player will start with the digits 1 to 9 available on the game board and will roll the dice to eliminate as many digits on the board as possible. The goal, as in golf, is to be the player with the lowest score at the end of the game.

- Each player should start with a game board and a writing utensil. The game board has the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 as well as space to write the score for each round.
- Players should decide on a player order for the game and give the dice to Player A.
- To start, Player A will roll the two dice. Whatever the numbers rolled, the player should calculate the sum. The player now should mark off any number of digits on his or her game board as long as they have the same sum as the roll. For example, if a player rolls a 1 & 5 for a sum of 6, he or she can mark off the following: 6; 1 and 5; 2 and 4 or 1, 2 and 3.
- Next, Player B repeats the process. The rest of the players should take their first turns. Then return to Player A for the second turn of the round.
- Player A now rolls again, but he or she has limited the options on the game board. Players are not allowed to reuse a digit that has already been eliminated and must work to find the sum by using the remaining numbers.
- First round play continues this way, but at any point, if a player is unable to make a sum from the remaining numbers, his or her round is over. This player will sit out play while the other players continue. Once every player's round has finished, each player should write his or her round score, the sum of the unmarked digits, on the game board.
- To begin the next round, players should clear their game board (erase all marks) making every digit available again. Follow the same steps as round 1. This process should be repeated for 18 rounds (or fewer depending on time constraints). The winner is the player with the lowest total score—the sum of all 18 round scores.

Optional: Make It Extra Mathy

Mathematical Exploration

Dice Roll Probability

At first glance, this may seem to be a game of pure chance. The sum students roll is the sum they eliminate. To develop a strategy for playing and to show that the game is not entirely up to chance, lead your students through the following exploration.

- ***In this game, you are always eliminating a sum equivalent to your dice roll. Is this purely a game of chance? Does it matter which numbers you choose for each sum?***

  Students might initially assume that this is a game of chance, but since our rules do not require the elimination of the exact two numbers rolled, but instead provide options, one can assume there is an element of control or strategy. Let's explore possible rolls and corresponding moves further.

- ***What are all the possible sums you can roll?***

  We know the lowest possible sum is 2, resulting from rolling two 1s. Similarly, the highest possible sum is 12, resulting from rolling two 6s. We can obtain all the sums in between by simply adding 1 repeatedly to either die from the roll 1 & 1. The possible sums are therefore 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

- ***Are each of theses sums equally likely to be rolled?***

  The easiest way to look at the probabilities is to create a chart of the two individual die rolls to see the frequency of occurrence of their sums (Figure 1). From the chart, we see that not all sums are equally likely. The probabilities of rolling sums of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 are 1/36, 2/36 or 1/18, 3/36 or 1/12, 4/36 or 1/9, 5/36, 6/36 or 1/6, 5/36, 4/36 or 1/9, 3/36 or 1/12, 2/36 or 1/18 and 1/36, respectively. Notice that the median sum, 7, is the most likely, and each sum gets less likely as we move +/−1 away from 7 until reaching 2 and 12, the least likely.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Figure 1

- ***How does this influence your strategy? Is there anything else you should be looking at that might be helpful in understanding this game?***

  Just by looking at the probabilities of sums, students may assume the best strategy would be to eliminate numbers like 2, 3 or 4 since they are less likely to be rolled than 5, 6, 7, 8 or 9. However, this isn't necessarily the case, as shown below. They should also look at all of the combinations of the digits 1 through 9 that can create each of the eleven sums. Once they do this, they will have a better picture with which to strategize.

Arithmetic, Combinations and Game Strategy

A more in-depth analysis of this game requires us to also look at the numbers of combinations of the digits presented on the game board that sum to 2 through 12. Once students analyze this, they can make better conclusions and decisions for playing the game strategically.

- ***How many ways are there to create unique combinations of 1 to 9 for each sum?***

  Figure 2 has all the possible combinations of digits 1 to 9 that sum to the quantities 2 to 12. We notice that for the quantities 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12, there are, respectively, 1, 2, 2, 3, 4, 5, 6, 8, 9, 10 and 12 ways to use the numbers 1 through 9 to achieve the specified sum. (A digit occurs only once in each sum, as in the game.)

- ***What observations can you make by looking at this list of sums? How could this influence your strategy? (Hint: look at the occurrences of each digit.)***

  Two possible observations are (1) the larger the sum, the more possible digit combinations and (2) the greater the digit, the less it appears in possible combinations (Figure 3). These observations could lead students to eliminate higher numbers first, but there is still something else to be looked at!

- ***Say you were to be left with three digits on your game board. Are there certain digits that put you in a more advantageous position?***

  If you used the strategy of eliminating the highest numbers first, you might think the best scenario is to have 1, 2 and 3 left. Then you can create the sums 2, 3, 4 and 5. This gives you a probability of 1/36 + 2/36 + 3/36 + 4/36 = 10/36 that your next roll will allow you to make a move. However, if you look at other combinations, you can find a better situation to be in. For example, if you have the numbers 3, 4 and 5, you can create the sums 3, 4, 5, 7, 8, 9 and 12. The probability of being able to make a move on the next roll is therefore 2/36 + 3/36 + 4/36 + 6/36 + 5/36 + 4/36 + 1/36 = 25/36. You are more likely than not to be able to make a move. This is not the only three-digit combination with a greater than 50% chance. Have your students find some others! (*Note:* These include 1, 2 and 4; 1, 2 and 5;1, 2 and 6; 2, 3 and 4; 2, 4 and 5; 4, 5 and 6.)

- ***What are some final takeaways from this?***

  In general, you can see that lower digits might be more desirable to remain on the board, but you must consider this within the context of possible combinations of digits and roll probabilities. For each move, you should reconsider your options and what is left on the board. What combinations can still be made? Would a certain move provide a better probability for the next? Determining the number of possible game outcomes gets very complicated, so it is nearly impossible to develop a foolproof strategy. However knowing how to calculate or estimate probabilities along the way will help you to win!

Sum	1#	2#'s	3#'s	4#'s	Total Ways
2	2	—	—	—	1
3	3	1+2	—	—	2
4	4	1+3	—	—	2
5	5	1+4 2+3	—	—	3
6	6	1+5 2+4	1+2+3	—	4
7	7	1+6 2+5 3+4	1+2+4	—	5
8	8	1+7 2+6 3+5	1+2+5 1+3+4	—	6
9	9	1+8 2+7 3+6 4+5	1+2+6 1+3+5 2+3+4	—	8
10	—	1+9 2+8 3+7 4+6	1+2+7 1+3+6 1+4+5 2+3+5	1+2+3+4	9
11	—	2+9 3+8 4+7 5+6	1+2+8 1+3+7 1+4+6 2+3+6 2+4+5	1+2+3+5	10
12	—	3+9 4+8 5+7	1+2+9 1+3+8 1+4+7 1+5+6 2+3+7 2+4+6 3+4+5	1+2+3+6 1+2+4+5	12

Figure 2

Digit	Uses
9	5
8	7
7	10
6	13
5	16
4	19
3	23
2	26
1	28

Figure 3

Differentiation, Scaling and Extensions

Change the Difficulty Level

If you have students who need an easier or harder version of the game, there are a few simple ways to change the game to meet their needs. Possible ways to scale Math Golf include:

- To make it easier, play with only the digits 1 to 6 or allow students to keep the sums chart in front of them to help them make decisions as the play.
- To make it harder, add more numbers to the game board and/or put more dice in play or introduce multiplication instead of or along with addition.

Extend Student Understanding

Challenge students to extend their understanding of the concepts behind this game by changing the rules and seeing if they can adjust their strategy. Perhaps the highest score rather than the lowest might win, or the round score might be the number of digits left (not the sum of the values).

MATH GOLF

1	2	3
4	5	6
7	8	9

MY SCORE:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18