

July 8
MATHCLUB Day
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MATH CLUB PROBLEM SET

1) Maricela distributed x gold pencils evenly among 17 gift bags for her Gold Level club members such that every bag received the greatest possible number of pencils, but some pencils were left over. What is the greatest possible number of pencils that could have been left over?



6) If $x \oplus y$ is defined as $xy + (2x - y) + y^2$, what is the value of $20 \oplus 6$?



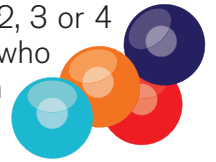
7) Working together on their Gold Level project, 2 students can find 8 scavenger hunt items in 3 hours. How many hours would it take 3 students to find 12 scavenger hunt items so their club can achieve Gold Level?

2) If all the letters of the word CLUB are used, in how many different ways can the four letters be arranged in a four-letter sequence?



3) Rani's math club T-shirt design has the program logo—a regular hexagon—inscribed in a circle of radius 7 inches. What is the perimeter of the hexagonal program logo?

8) Ron and Martin are playing the Marble Challenge game. They have a bowl containing 39 marbles. Each player takes turns removing 1, 2, 3 or 4 marbles from the bowl. The person who removes the last marble loses. If Ron takes the first turn to start the game, how many marbles should he remove to guarantee he is the winner?

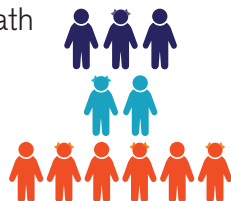


4) During Dice Duel, Amir rolls a fair 8-sided die with faces numbered 1 through 8 four times. What is the probability that Amir rolls exactly one 3, and that the 3 is rolled on the third roll? Express your answer as a common fraction.

9) Ms. Reyes divides her club into 2 teams for Talk Like a Pirate Day. The Buccaneers Team and the Green Parrots Team start the activity at the same time. The Buccaneers finish in 34 minutes and 27 seconds. The Green Parrots finish in 35 minutes and 25 seconds. How many seconds ahead of the Green Parrots do the Buccaneers finish?



5) The ratio of 6th graders to 7th graders to 8th graders in Mr. Babson's math club is 3:2:6. If there are 33 total students in Mr. Babson's math club, how many are 8th graders?



10) Janet and Ronan are playing Reflection Battle with a sheet of graph paper. They draw a coordinate grid on the paper and plot the line $y = x$ as their line of symmetry. If Ronan plots the point $(-6, 14)$, what point must Janet plot to reflect Ronan's point over the line of symmetry?

11) Northwood Middle School's math club is trying to schedule their math club meetings for the year. One possibility is for the club to meet every day from 10:00 am to 11:00 am. Another possibility is meeting from 10:00 am to 11:15am each of four days. A third possibility is to meet from 10:00 am to 11:40 am each of three days. The club may meet on any of the five days of the school week. In how many different ways could the club meet?

16) A regular hexagonal table that is by itself seats 6 people, one person at each side. A row of these hexagonal tables is created by pushing together a certain number of them so that a side of one table meets a side of the next table. If 50 mathletes can sit at the row of tables that was created for their math club meeting, how many tables are in the row?



12) Ms. Grady is applying for Silver Level and is trying to figure out how often her club met over the course of November, December and January. During the first 2 months, the club met, on average, once every 5 days. If the club met 22 times in total, on average, how often did they meet in the third month? Round your answer to the nearest whole number of days.

17) For a certain move on his A-maze-ing Fractions board, Estephan accidentally switched the whole number part and the denominator of his mixed number. When he subtracted his new mixed number from the correct mixed number, his answer was $1 \frac{1}{6}$. If both mixed numbers had a one in the numerator, what was the original mixed number?

13) Marco and Trevor are playing Make 100. Six cards are chosen at random from a standard deck of 52 cards. What is the probability that all six cards are the same suit? Express your answer as a common fraction.



14) After doing the Make Your Money Grow activity with her math club, Tishe was inspired to invest her money. She deposited \$200 into an account with 8% interest compounded annually for 3 years. What is the number of dollars in the amount of interest Tishe earned? Express your answer as a decimal to the nearest cent.

18) Mr. Scott drives to his weekly math club meeting. Half the time during his 25-mile trip, his car averages 45 mi/h. Given that the trip takes 40 minutes, how many mi/h was his average speed for the rest of the trip?



19) DeShawn finds that his foot is 21 M&Ms long in the Measure Your Feet Day problem set. If the length of 5 paper clips is equal to the length of 7 pennies, and the length of 2 pennies is equal to the length of 3 M&Ms, how long is DeShawn's foot in paper clips?

15) Eisha rolls a product of 24 on her first turn in Fence Me In, which means she must make a rectangle with an area of 24 units². She wants her rectangle to have a length that is six times its width. What is the perimeter of Eisha's rectangle?

20) For Bake Cookies Day, Ms. Thibodeaux wants to actually bake cookies with her club after they complete the problem set. If the club uses a pan that is 12 inches by 15 inches, and the cookies will each have a radius of exactly 1.5 inches when cooked, what is the maximum number of cookies they can put on this pan to cook?

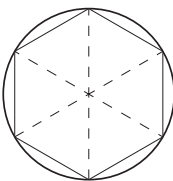


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MATH CLUB PROBLEM SET SOLUTIONS

1) The greatest possible remainder when dividing by 17 is **16**. There could be 16 pencils left over. If there were 17 or more pencils left over, then Marcela would have added another pencil to each gift bag.

2) Students may have realized there are $4! = \mathbf{24}$ ways to arrange four different letters. Alternatively, you could list all of these possibilities: CLUB, CLBU, CUBL, CULB, CBUL, CBLU, BCLU, BCUL, BLUC, BLCU, BUCL, BULC, LUCB, LUBC, LBCU, LBUC, LCUB, LCBU, UBLC, UBCL, ULCB, ULBC, UCLB and UCBL. Note, after listing the combinations starting with C, we can see that there are 6 possibilities; we know, therefore, that we'll have 6 possible combinations when each of the remaining letters is listed first.

3) The hexagon can be subdivided into 6 equilateral triangles, as shown below. If the radius of the circle is 7 inches, then each side of the 6 equilateral triangles is 7 inches. Now we can see the perimeter of the hexagon much be $6 \times 7 = \mathbf{42}$ inches.



4) The first, second and fourth rolls can be any number other than 3 and the third roll must be a 3, so the probability that the only 3 occurs on the third roll is $(7/8) \times (7/8) \times (1/8) \times (7/8) = \mathbf{343/4096}$.

5) Since the ratio of 6th graders to 7th graders to 8th graders is 3 to 2 to 6, that means there are $3 + 2 + 6 = 11$ equal "parts" of the club, and that 6 of those "parts" are 8th graders. Because we are told the total number of students in the club, we can set up a proportion such that $6/11 = x/33 \rightarrow 6(33) = 11x \rightarrow 198 = 11x \rightarrow x = \mathbf{18}$ 8th graders.

6) The hexagon is a made-up operation designed for this problem. Substituting 20 for x and 6 for y in the definition, we get $20(6) + (2(20) - 6) + 6^2 \rightarrow 120 + (40 - 6) + 36 \rightarrow 120 + 34 + 36 = \mathbf{190}$.

7) If working together, 2 students can find 8 scavenger hunt items in 3 hours, then each of the students, working alone, can find 4 scavenger hunt items in 3 hours. Therefore, 3 students can find $3 \times 4 = 12$ scavenger hunt items in the same **3** hours.

8) Ron should take **3** marbles to leave Martin with 36 marbles. To explain this, we will work backward. At the end of the game, Ron wants to leave 1 marble so Martin will have to take the last marble. Thus, Martin will lose. To ensure that he can leave Martin with 1 marble, Ron should leave 6 marbles before that. By doing so, Ron can ensure that 5 marbles are removed between his turn and Martin's final turn. For example, if Martin takes 1 marble, Ron takes 4 marbles (or vice-versa). Or, if Martin takes 2, Ron takes 3 (or vice-versa). Therefore, as long as Ron leaves Martin with a number that is 1 more than a multiple of 5, Ron will completely control the game.

9) The Buccaneers finish in 34 minutes and 27 seconds. If the Green Parrots finished in 35 minutes and 27 seconds they would finish 1 minute, or 60 seconds, later than the Buccaneers. As it turns out, the Green Parrots actually finish 2 seconds faster than that, so the Buccaneers finish **58** seconds ahead of the Green Parrots.

10) If the indicated line of symmetry is $y = x$, then Ronan's plotted y coordinate (14) must equal the x coordinate Janet plots and Ronan's plotted x coordinate (-6) must equal the y coordinate Janet plots. So, Janet must plot point **(14, -6)**.

11) There is only 1 way for the club to meet every day. If the club meets on each of four days, there are 5 ways to schedule the meetings. The day off could be any of Monday, Tuesday, Wednesday, Thursday or Friday. Finally, there are 10 ways to choose 2 days off in order for the club to meet each of three days (i.e. ${}_5C_2 = 5!/(2!3!) = (5 \times 4)/(2 \times 1) = 10$). In total, there are $1 + 5 + 10 = \mathbf{16}$ ways to schedule the math club meetings.

12) November has 30 days and December has 31 days, so in the first two months, the club met $(30 + 31)/5 = 12.2$ times. This means that in the third month, the club met $22 - 12.2 = 9.8$ times. Since January has 31 days, the club met, on average, every $31/9.8 \approx 3$ days.

13) The probability of the first card being a heart, for example, is $13/52$. The probability of the second card being a heart is then $12/51$, since the pool of hearts and the total number of cards is now one less. Continuing in this way, the probability of the six cards being the same suit is $13/52 \times 12/51 \times 11/50 \times 10/49 \times 9/48 \times 8/47$. By canceling common factors, we get $(11 \times 9 \times 8)/(4 \times 51 \times 5 \times 49 \times 4 \times 47) = (11 \times 9 \times 2)/(4 \times 51 \times 5 \times 49 \times 47) = 198/2,349,060 = 66/783,020$. Because this is possible for each of the four suits, we must divide the denominator by 4 to represent the probability of this happening with a single suit. Thus, the probability is **66/195,755**.

14) After the first year, Tishe will have $200(1.08) = \$216$. After the second year, Tishe will have $216(1.08) = \$233.28$. After the third year, Tishe will have $233.28(1.08) = \$251.94$. So, the amount of interest Tishe earned is $\$251.94 - \$200 = \mathbf{\$51.94}$. Alternatively, you could shorten this process by solving $200(1.08)^3 = \$251.94$.

15) If the area of the rectangle is 24 units² and the length of the rectangle must be six times its width, then we need to find two factors of 24 that satisfy this requirement. We know that $(6w)(w) = 6w^2 = 24$, which means $w = 2$ units. If the width is 2 units, then the length must be $6(2) = 12$ units. Thus, the perimeter of Eisha's rectangle is $12 + 12 + 2 + 2 = \mathbf{28}$ units.

16) The hexagonal tables that are on the ends seat 5 people each. The other $50 - 2 \times 5 = 40$ people are seated at hexagonal tables that are in the middle of the row and seat only 4 people each. Thus, $40 \div 4 = 10$ more tables are needed in between the two end tables, which is a total of $2 + 10 = \mathbf{12}$ tables.

17) Because the answer is $1 \frac{1}{6}$ when he subtracts his new mixed number from the correct mixed number, we can assume that the swapped values are consecutive numbers. Because we know both values are mixed numbers and they both have a numerator of 1, if we were to try 1 and 2 as our swapped values, the original mixed number would be $2 \frac{1}{1} = 2$, which is not a mixed number. Let's try 2 and 3 instead. In this case, our subtraction problem would look like $3 \frac{1}{2} - 2 \frac{1}{3} = 1 \frac{1}{6}$, so we know that 2 and 3 are the correct swapped values and the original mixed number is **$3 \frac{1}{2}$** .

18) If Mr. Scott drove for half the time at a speed of 45 mi/h, then he drove at this speed for $40/2 = 20$ minutes, which is $1/3$ of an hour. So, we can find the distance over which he was going this speed by using the distance formula, $d = rt$: $d = 45(1/3) = 15$ miles. This means that the remaining distance, $25 - 15 = 10$ miles, was traveled for the remaining 20 minutes of the drive (or $1/3$ hour). We can find the rate at which he drove during this time by again using the distance formula: $d = rt \rightarrow 10 = r(1/3) \rightarrow r = \mathbf{30}$ mi/h.

19) If the ratio of pennies to M&Ms is 2:3, we can scale this ratio up by a factor of 7 to get 14:21. So, the length of 21 M&Ms is the same length as 14 pennies. Now, if we know that the ratio of paper clips to pennies is 5:7, we can scale this ratio up by a factor of 2 to get 10:14. So, the length of 14 pennies is the same length as 10 paper clips. Because the length of 14 pennies is the same length as both 21 M&Ms and 10 paper clips, we can say that 21 M&Ms is the same length as 10 paper clips. So, DeShawn's foot is **10** paper clips long.

20) If each cookie will have a radius of 1.5 inches, this means they will each have a diameter of 3 inches. The pan could fit 4 3-inch cookies across the 12-inch side, and 5 3-inch cookies across the 15-inch side. So, the maximum number of cookies the pan could fit is $4 \times 5 = \mathbf{20}$ cookies.