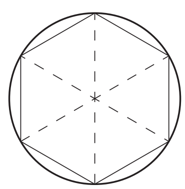
July 8 MATHCLUB Day powered by MATHCOUNTS®

MATH CLUB PROBLEM SET SOLUTIONS

1) The greatest possible remainder when dividing by 17 is **16**. There could be 16 pencils left over. If there were 17 or more pencils left over, then Maricela would have added another pencil to each gift bag.

2) Students may have realized there are $4! =$ **24** ways to arrange four different letters. Alternatively, you could list all of these possibilities: CLUB, CLBU, CUBL, CULB, CBUL, CBLU, BCLU, BCUL, BLUC, BLCU, BUCL, BULC, LUCB, LUBC, LBCU, LBUC, LCUB, LCBU, UBLC, UBCL, ULCB, ULBC, UCLB and UCBL. Note, after listing the combinations starting with C, we can see that there are 6 possibilities; we know, therefore, that we'll have 6 possible combinations when each of the remaining letters is listed first.

3) The hexagon can be subdivided into 6 equilateral triangles, as shown below. If the radius of the circle is 7 inches, then each side of the 6 equilateral triangles is 7 inches. Now we can see the perimeter of the hexagon much be $6 \times 7 =$ **42** inches.



4) The first, second and fourth rolls can be any number other than 3 and the third roll must be a 3, so the probability that the only 3 occurs on the third roll is $(7/8) \times (7/8) \times (1/8) \times (7/8) =$ **343/4096**.

5) Since the ratio of 6th graders to 7th graders to 8th graders is 3 to 2 to 6, that means there are $3 + 2 + 6 = 11$ equal "parts" of the club, and that 6 of those "parts" are 8th graders. Because we are told the total number of students in the club, we can set up a proportion such that $6/11 = x/33 \rightarrow 6(33) = 11x \rightarrow 198 = 11x \rightarrow x =$ **18** 8th graders.

6) The hexagon is a made-up operation designed for this problem. Substituting 20 for x and 6 for y in the definition, we get $20(6) + (2(20) - 6) + 6^2 \rightarrow 120 + (40 - 6) + 36 \rightarrow 120 + 34 + 36 =$ **190**.

7) If working together, 2 students can find 8 scavenger hunt items in 3 hours, then each of the students, working alone, can find 4 scavenger hunt items in 3 hours. Therefore, 3 students can find $3 \times 4 = 12$ scavenger hunt items in the same **3** hours.

8) Ron should take **3** marbles to leave Martin with 36 marbles. To explain this, we will work backward. At the end of the game, Ron wants to leave 1 marble so Martin will have to take the last marble. Thus, Martin will lose. To ensure that he can leave Martin with 1 marble, Ron should leave 6 marbles before that. By doing so, Ron can ensure that 5 marbles are removed between his turn and Martin's final turn. For example, if Martin takes 1 marble, Ron takes 4 marbles (or vice-versa). Or, if Martin takes 2, Ron takes 3 (or vice-versa). Therefore, as long as Ron leaves Martin with a number that is 1 more than a multiple of 5, Ron will completely control the game.

9) The Buccaneers finish in 34 minutes and 27 seconds. If the Green Parrots finished in 35 minutes and 27 seconds they would finish 1 minute, or 60 seconds, later than the Buccaneers. As it turns out, the Green Parrots actually finish 2 seconds faster than that, so the Buccaneers finish **58** seconds ahead of the Green Parrots.

10) If the indicated line of symmetry is $y = x$, then Ronan's plotted y coordinate (14) must equal the x coordinate Janet plots and Ronan's plotted x coordinate (-6) must equal the y coordinate Janet plots. So, Janet must plot point **(14, −6)**.

11) There is only 1 way for the club to meet every day. If the club meets on each of four days, there are 5 ways to schedule the meetings. The day off could be any of Monday, Tuesday, Wednesday, Thursday or Friday. Finally, there are 10 ways to choose 2 days off in order for the club to meet each of three days (i.e. ${}_5C_2 = 5!/(2!3!) = (5 \times 4)/(2 \times 1) = 10$). In total, there are $1 + 5 + 10 =$ **16** ways to schedule the math club meetings.

12) November has 30 days and December has 31 days, so in the first two months, the club met $(30 + 31)/5 = 12.2$ times. This means that in the third month, the club met $22 - 12.2 = 9.8$ times. Since January has 31 days, the club met, on average, every $31/9.8 \approx$ **3** days.

13) The probability of the first card being a heart, for example, is 13/52. The probability of the second card being a heart is then 12/51, since the pool of hearts and the total number of cards is now one less. Continuing in this way, the probability of the six cards being the same suit is $13/52 \times 12/51 \times 11/50 \times 10/49 \times 9/48 \times 8/47$. By canceling common factors, we get $(11 \times 9 \times 8)/(4 \times 51 \times 5 \times 49 \times 4 \times 47) = (11 \times 9 \times 2)/(4 \times 51 \times 5 \times 49 \times 47) = 198/2{,}349{,}060 = 66/783{,}020$. Because this is possible for each of the four suits, we must divide the denominator by 4 to represent the probability of this happening with a single suit. Thus, the probability is **66/195,755**.

14) After the first year, Tishe will have 200(1.08) = \$216. After the second year, Tishe will have 216(1.08) = \$233.28. After the third year, Tishe will have 233.28(1.08) = \$251.94. So, the amount of interest Tishe earned is \$251.94 – \$200 = **\$51.94**. Alternatively, you could shorten this process by solving $200(1.08)^3 = \$251.94$.

15) If the area of the rectangle is 24 units^2 and the length of the rectangle must be six times its width, then we need to find two factors of 24 that satisfy this requirement. We know that $(6w)(w) = 6w^2 = 24$, which means $w = 2$ units. If the width is 2 units, then the length must be 6(2) = 12 units. Thus, the perimeter of Eisha's rectangle is 12 + 12 + 2 + 2 = **28** units.

16) The hexagonal tables that are on the ends seat 5 people each. The other $50 - 2 \times 5 = 40$ people are seated at hexagonal tables that are in the middle of the row and seat only 4 people each. Thus, $40 \div 4 = 10$ more tables are needed in between the two end tables, which is a total of 2 + 10 = **12** tables.

17) Because the answer is 1 1/6 when he subtracts his new mixed number from the correct mixed number, we can assume that the swapped values are consecutive numbers. Because we know both values are mixed numbers and they both have a numerator of 1, if we were to try 1 and 2 as our swapped values, the original mixed number would be 2 1/1 = 2, which is not a mixed number. Let's try 2 and 3 instead. In this case, our subtraction problem would look like 3 ½ – 2 1/3 = 1 1/6, so we know that 2 and 3 are the correct swapped values and the original mixed number is **3 ½**.

18) If Mr. Scott drove for half the time at a speed of 45 mi/h, then he drove at this speed for 40/2 = 20 minutes, which is 1/3 of an hour. So, we can find the distance over which he was going this speed by using the distance formula, $d = rt$: $d = 45(1/3) = 15$ miles. This means that the remaining distance, 25 – 15 = 10 miles, was traveled for the remaining 20 minutes of the drive (or 1/3 hour). We can find the rate at which he drove during this time by again using the distance formula: $d = rt \rightarrow 10 = r(1/3) \rightarrow r =$ **30** mi/h.

19) If the ratio of pennies to M&Ms is 2:3, we can scale this ratio up by a factor of 7 to get 14:21. So, the length of 21 M&Ms is the same length as 14 pennies. Now, if we know that the ratio of paper clips to pennies is 5:7, we can scale this ratio up by a factor of 2 to get 10:14. So, the length of 14 pennies is the same length as 10 paper clips. Because the length of 14 pennies is the same length as both 21 M&Ms and 10 paper clips, we can say that 21 M&Ms is the same length as 10 paper clips. So, DeShawn's foot is **10** paper clips long.

20) If each cookie will have a radius of 1.5 inches, this means they will each have a diameter of 3 inches. The pan could fit 4 3-inch cookies across the 12-inch side, and 5 3-inch cookies across the 15-inch side. So, the maximum number of cookies the pan could fit is $4 \times 5 =$ **20** cookies.