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Art of Problem Solving
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*Activity Sheet for the March, 2020, MATHCOUNTS Mini*


Try these problems before watching the lesson.

1. What is the ratio of the volume of a cube with edge length six inches to the volume of a cube with edge length one foot? Express your answer as a common fraction.

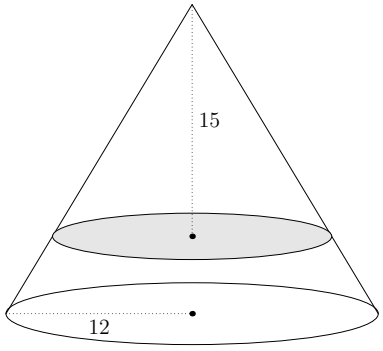
2. The height of a right circular cone is three times its radius. If the circumference of the base of the cone is $6\pi$, what is the volume of the cone?

3. Suppose $\overline{AB}$, $\overline{AC}$, and $\overline{AD}$ are edges of a cube that has side length 6 cm. What is the volume of tetrahedron $ABCD$?

4. P.J. has a cylindrical mug with a 3-inch diameter. His mug is filled to a height of 6 inches with grape juice. Schuyler has a cylindrical mug with a 4-inch diameter. To what height in inches must Schuyler's mug be filled so that he receives the same amount of juice as P.J.? Express your answer as a mixed number.

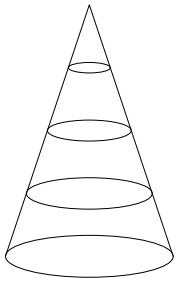

**First Problem:** A regular tetrahedron is a solid with four equilateral triangular faces. What is the height of a regular tetrahedron with edges of length 5 cm? Express your answer as a common fraction in simplest radical form.

**Second Problem:** A cone with base radius 12 cm is sliced parallel to its base, as shown, to remove a smaller cone of height 15 cm. If the height of the smaller cone is three-fourths that of the original cone, what is the volume of the remaining frustum? Express your answer in terms of $\pi$.




## Follow-up Problems

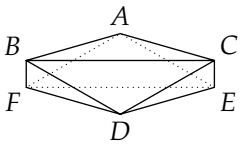
5. A right circular cone is sliced into four pieces by planes parallel to its base, as shown in the figure. All of these pieces have the same height. What is the ratio of the volume of the second-largest piece to the volume of the largest piece? Express your answer as a common fraction.



6. Two cross sections of a right hexagonal pyramid are obtained by cutting the pyramid with planes parallel to the hexagonal base. The areas of the cross sections are $216\sqrt{3}$ square feet and $486\sqrt{3}$ square feet. The two planes are 8 feet apart. How far from the apex of the pyramid is the larger cross section?

7. A right cylindrical oil tank is 15 feet tall and its circular bases have diameters of 4 feet each. When the tank is lying flat on its side (not on one of the circular ends), the oil inside is 3 feet deep. How deep, in feet, would the oil have been if the tank had been standing upright on one of its bases? Express your answer as a decimal to the nearest tenth.

8. Faces $ABC$ and $DEF$ of the polyhedron below are parallel equilateral triangles with side length $4\sqrt{2}$ units. Each of the other edges in the polyhedron has length 4 units (i.e. $AE = EC = CD = DB = BF = FA = 4$). Find the volume of the polyhedron.




## Wow! Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).