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Activity Sheet for the March, 2019, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

- 1. In  $\triangle XYZ$ , we have  $\angle X = 30^\circ$ ,  $\angle Y = 90^\circ$ , and YZ = 6. Find XY and XZ.
- 2. What is the area of an equilateral triangle with side length 6?
- 3. What is the area, in square feet, of an isosceles triangle whose vertex angle is 120° and whose base is 20 feet long? Express your answer as a common fraction in simplest radical form.
- 4. Billy is hiking in Colorado. He walks eastward four miles, then turns 60 degrees northward and walks six miles. How far is he from his starting point? Express your answer in simplest radical form.

1 The Problems

**First Problem:** Congruent, non-overlapping circles A, B, C and D are positioned in a plane, such that A, B and C are mutually tangent to each other, and circle D is tangent to circle C. Triangle EFG circumscribes the four circles as shown. What is the length of side FG?





**Second Problem:** A unit square contains four congruent non-overlapping equilateral triangles as shown in the figure. What is the largest possible side-length of one of the triangles?





- 5. The altitudes of equilateral triangle ABC all pass through point H inside the triangle. If AB = 6, then what is AH?
- 6. Six circles of radius 1 unit are drawn tangent to the sides of a regular hexagon. Each circle is also tangent to two other circles as shown in the drawing. What is the perimeter of the hexagon? Express your answer in simplest radical form.



7. Two angles of a triangle measure 30 and 45 degrees. If the side of the triangle opposite the 30-degree angle measures  $6\sqrt{2}$  units, what is the sum of the lengths of the two remaining sides? Express your answer as a decimal to the nearest tenth.



8. In triangle ABC,  $m \angle ABC = 120^{\circ}$ , AB = 3 and BC = 4. If lines perpendicular to  $\overline{AB}$  at point A and to  $\overline{BC}$  at point C meet at point D, then find CD. (Source: AMC 12)



Share Your Thoughts Wow

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