



Try these problems before watching the lesson.

1. If $a \textcircled{C} b = a^2 + ab + b^2$, what is the value of $(3 \textcircled{C} 4) - 12$?
2. If $f(x) = x^2 - 3$ and $g(x) = 2x + 1$, what is $f(g(3))$?
3. What single digit does D represent when $2D \times D51 = 807D$?
4. A sequence of numbers, $1, 1, 3, \dots$, is formed with the rule $A_{n+2} = A_{n+1} + 2A_n$. What is the sixth term?



Let $x \star y = x^2 + xy$. If a and b are positive integers such that $a \star b = 9$ and $b \star a = 72$, what is the value of $\frac{a}{b}$? Express your answer as a common fraction.

 Follow-up Problems

5. Suppose that $A \& B = k \times A^m \times B^n$, where k , m and n are constants. Suppose that $5 \& 3 = 18$, $10 \& 3 = 72$ and $5 \& 6 = 36$. What is the value of $10 \& 6$?

6. If $p \oplus q = \sqrt{p^2 + q^2}$ and $p \ominus q = \sqrt{p^2 - q^2}$, what is the value of


$$(1 \oplus 2 \oplus 3 \oplus 4 \oplus 5 \oplus 6 \oplus 7 \oplus 9 \oplus 10) \ominus (1 \oplus 2 \oplus 3 \oplus 4 \oplus 5 \oplus 6 \oplus 7 \oplus 8 \oplus 9)?$$

7. The n th term of a sequence is $a_n = (-1)^{n+1}(3n + 2)$. What is the value of

$$a_1 + a_2 + \cdots + a_{100}?$$

8. Let $f(n) = \begin{cases} n^2 + 1 & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$.

For how many integers n from 1 to 100, inclusive, does $f(f(\cdots f(n)\cdots)) = 1$ for some number of applications of f ?

 Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).