Fun Problem-Solving Techniques

Fun, hands-on ways to solve math problems

Being able to solve different problems in a variety of ways is an invaluable skill. Luckily, it can be really fun to practice problem-solving techniques! There are a variety of math topics covered in the problems used for this activity. However, there is a focus on problems that can be solved by creating a model, acting out a situation, drawing a picture or making a list.

The problems used in this activity do not require any advanced math. Students may work independently or in groups on the following four sets of problems. We recommend having students share their solutions and creations for one part of the problem set before beginning work on the next part.

WHY CLUB LEADERS & KIDS LOVE IT

- Students have the opportunity to engage in hands-on learning, making problems feel less like "textbook" math.
- Students boost their confidence by tackling challenging math problems in an accessible way.
- Setup is easy, and there are not many handouts that the students need.

MATERIALS NEEDED

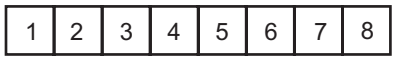
- 1 problem set for each student*
- Paper
- Scissors, rulers and tape
- Colored pencils (red, green and yellow)

*Included in this *Club Activity Book* and available for download at www.mathcounts.org/clubleaders

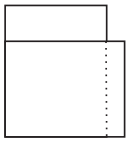
PROBLEM-SOLVING TECHNIQUE 1: CREATE A MODEL

For the first problem set, "Create a Model," explain to students that you would like them to create the actual model for each problem while they are solving it. Students will need pieces of paper, rulers, tape and scissors.

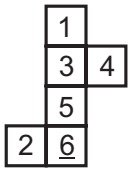
Problem Set Part 1:



1. A strip of paper consists of eight squares as shown. The strip is folded in half so that the rightmost square (#8) lands facedown and on top of the leftmost square (#1). Then, the new rightmost square is again folded over on top of the new leftmost square, and then again one more time, so the strip has been folded into a stack of single squares with the square labeled 1 still on the bottom. What number is in the square on the top of the stack? *(2006 State Sprint Round)*



2. A sheet of 8-inch by 10-inch paper is placed on top of a sheet of 8.5-inch by 11-inch paper, as shown. What is the area of the region of overlap, in square inches? *(2005 School Sprint Round)*



3. This figure to the left is folded into a cube. When the cube is rolled, the lateral product is the product of the numbers on the four lateral faces. The numbers on the top and bottom faces are not included in the multiplication. What is the greatest possible lateral product for this cube? *(2004 Chapter Sprint Round)*

PROBLEM-SOLVING TECHNIQUE 2: ACT OUT A SITUATION

For the second part of the problem set, "Act Out a Situation," explain to students that they should actually perform the scenarios in order to answer the questions. Students will need pieces of paper, rulers and scissors.

Problem Set Part 2:

4. An arm-wrestling tournament begins with 5 people. When 2 people arm wrestle (a duel), the loser is eliminated from the tournament. Eventually, only 1 person remains as the tournament winner. How many arm-wrestling duels have been completed when the tournament winner is determined? (*Modified from 2007–2008 MATHCOUNTS School Handbook, Volume II)*

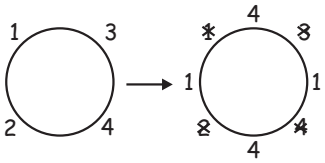
5. A rectangular candy wrapper is made from a 1-inch by 3-inch piece of paper. What is the greatest number of wrappers that can be cut from a rectangular piece of paper measuring 8 inches by 9 inches? *(Modified from 2007–2008 MATHCOUNTS School Handbook, Volume I)*

6. When a piece of paper is folded in half, there are two layers of paper. How many layers will there be when a piece of paper is folded in half a total of six times without ever unfolding it? *(Modified from 2007–2008 MATHCOUNTS School Handbook, Volume I)*

PROBLEM-SOLVING TECHNIQUE 3: DRAW A PICTURE

For the third part of the problem set, "Draw a Picture," explain to students that they have to provide a picture that would assist in solving the problem. Students will need colored pencils.

Problem Set Part 3:

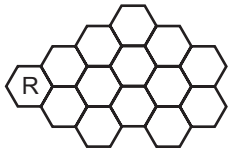


7. The numbers 1, 2, 3 and 4 are placed in any order about a circle. At each turn of a game, a new circle is formed with four new entries, each of which is the square of the difference of each pair of adjacent numbers in the previous circle. Each new entry is placed between the two numbers from which it was calculated, and the old numbers are erased. What is the largest possible number ever to appear when playing the game if any initial ordering of the numbers 1, 2, 3 and 4 may be used? (*2007–2008 MATHCOUNTS School Handbook, Volume II)*

8. How many diagonals does a regular octagon have? *(2005–2006 MATHCOUNTS School Handbook)*

9. Roslyn has 10 boxes. Exactly 5 of the boxes contain pencils, exactly 4 of the boxes contain pens and exactly 2 of the boxes contain both pens and pencils. How many boxes contain neither pencils nor pens? *(2005 Chapter Sprint Round)*

10. The hexagon with the "R" is colored red. Each hexagon is colored red, yellow or green, so that no two hexagons with a common side are the same color. In how many different ways can the figure be colored? *(2006 Chapter Sprint Round)*



PROBLEM-SOLVING TECHNIQUE 4: MAKE A LIST

For the fourth part, "Make a List," explain to students that they need to show their lists for each problem. Students may want to use colored pencils.

Problem Set Part 4:

11. If all of the letters of the word *BEEP* are used, in how many different ways can the 4 letters be arranged in a 4-letter sequence? The 2 *E*s are indistinguishable, so *EEPB* should be counted only once since we would not be able to see a difference if the 2 *E*s were swapped. *(2007–2008 MATHCOUNTS School Handbook, Volume I)*

12. Chandra has 3 bowls. Each one is a different color (red, green and yellow). She also has exactly 1 glass the same color as each bowl. If she chooses a bowl and a glass from the cupboard, how many pairings are possible? One such pairing is a green bowl and a yellow glass. *(Modified from 2005–2006 MATHCOUNTS School Handbook)*

13. The single-digit prime numbers are 2, 3, 5 and 7. How many positive two-digit integers are there in which each of the two digits is prime? *(2005 School Sprint Round)*

Answers: **1)** 2; **2)** 68 inches^2; **3)** 144; **4)** 4 duels; **5)** 24 wrappers; **6)** 64 layers; **7)** 16,777,216; **8)** 20 diagonals; **9)** 3 boxes; **10)** 2 ways; **11)** 12 ways; **12)** 9 pairings; **13)** 16 integers