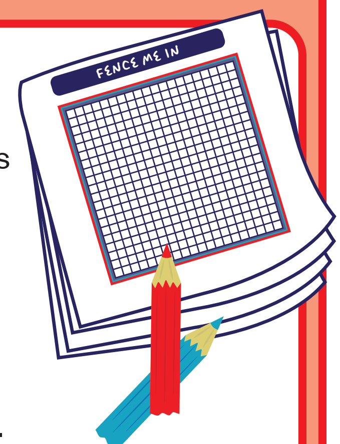
FENCE ME IN

EVERYTHING YOU NEED TO PLAY



MATERIALS

Fence Me In Game Board

- 1 game board per pair of students. These instructions assume that students are playing one on one, but students can also play in two teams. Find a copy of the game board below.
- Using a paper copy of the board is fine, but you may also want to make a laminated copy for multiple uses. A sheet protector is an easy substitute for lamination!

Colored Writing Utensils

- 1 per student—colored pencils or dry erase markers (for laminated boards). Make sure paired students have two different colors.

Standard Six-sided Dice

- 2 per pair of students.
- The MATHCOUNTS Club App is available in lieu of physical dice. If the app is used, one device is needed per pair of students.

RULES

The Fence Me In game board is a 20 × 20 square grid. Students will play one on one (or in two teams) and will try to cover the grid with squares and other rectangles. The size of a rectangle is, in part, determined by rolling two dice. The winner is the player with the most rectangles drawn.

- Decide who will be Player A and who will be Player B—rock-paper-scissors, flip a coin, etc.
- To start, Player A rolls the dice and multiplies the two numbers rolled. This is the “size” of the rectangle Player A must draw. Player A decides whether the product is the perimeter or the area of the rectangle. The player calculates the rectangle’s dimensions—they must be integers—and draws it anywhere on the grid and shades it in. For example, a product of 8 allows a rectangle of any of the following dimensions: for area, 1 × 8 or 2 × 4 and for perimeter, 1 × 3 or 2 × 2.
- Next, Player B rolls the dice and repeats the process. Player B’s rectangle can be anywhere on the grid but cannot overlap the area that Player A’s rectangle covers.
- The process is repeated, with each player in turn rolling the dice and drawing a rectangle. The game will begin to get challenging once the board starts filling up. If a player rolls the dice and is unable to draw a rectangle to fit the remaining space, he or she loses the turn.
- Players should continue taking turns until the board is completely filled or both players have rolled twice, consecutively, and were unable to draw a rectangle. In other words, if four straight rolls have been unsuccessful, the game is over.
- The winner, for this version, is the player with the most rectangles (see extensions section for rule variations). Because the board will start to become full of colored rectangles, it would be good to have players tally their own rectangles. This way it will be easy to determine the winner when the game ends.

Optional: Make It Extra Mathy

Mathematical Exploration

Working Backward with Area and Perimeter

Often students are given dimensions and are asked to calculate the area and perimeter. This game forces them to operate in reverse to find the dimensions when given the area or perimeter. This section explores what the formulas tell us about the dimensions students can select in this game and how to use the dimensions in their game-playing strategy.

- ***What are the formulas for area and perimeter? What does this tell you about numbers that can be used for area or perimeter?***

  Students are probably familiar with the formulas for area, $A = l \times w$, and perimeter, $P = 2l + 2w$, but based on these formulas, what observations can be made about the game? First, we know we can use area for any roll. This is because area is found by multiplying two dimensions, and when we roll the dice, we multiply the two numbers, so we are guaranteed to at least have those as options for the dimensions. Second, we see that the formula for perimeter can be factored and written as $P = 2(l + w)$. This tells us that since we are using integer dimensions, we can only use even product rolls for perimeter, since the product must be divisible by 2.

- ***If you roll the product 24 and choose to use perimeter, is the area of your rectangle the same for all possible dimensions? How would the area differ if you had used the 24 as your area?***

  Because the perimeter formula can be written $P = 2(l + w)$, the sum of our dimensions must be $24 \div 2 = 12$. Let's look at a couple of options. We could have dimensions of 1 and 11. This would give us an area of $1 \times 11 = 11$. We could also have dimensions of 6 and 6. This would give us an area of $6 \times 6 = 36$. So we can see that the area can be varied while maintaining the perimeter. This tells us that using perimeter in the game is a good strategy if you need a small rectangle to fit in a tight spot or if you want to take up a large area to block your opponent.

- ***How many unique rolls or products are possible?***

  Each die can have any of the numbers 1 through 6 rolled. The smallest possible product is $1 \times 1 = 1$ and the largest possible product is $6 \times 6 = 36$, but not all numbers between 1 and 36 can be rolled. Looking at the figure to the left, we can see that there are 18 possible products: 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30 and 36.

x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

- ***For each of these products, what are the possible dimensions?***

  Having students find all the possible dimensions, as in the chart at the right, will show them how different products have different numbers of possible dimensions. This would also be a helpful exercise, perhaps for

Possible Products and Dimensions

Product	Area Dimensions	Perimeter Dimensions
1	1×1	—
2	1×2	—
3	1×3	—
4	1×4; 2×2	1×1
5	1×5	—
6	1×6; 2×3	1×2
8	1×8; 2×4	1×3; 2×2
9	1×9; 3×3	—
10	1×10; 2×5	1×4; 2×3
12	1×12; 2×6; 3×4	1×5; 2×4; 3×3
15	1×15; 3×5	—
16	1×16; 2×8; 4×4	1×7; 2×6; 3×5; 4×4
18	1×18; 2×9; 3×6	1×8; 2×7; 3×6; 4×5
20	1×20; 2×10; 4×5	1×9; 2×8; 3×7; 4×6; 5×5
24	2×12; 3×8; 4×6	1×11; 2×10; 3×9; 4×8; 5×7; 6×6
25	5×5	—
30	2×15; 3×10; 5×6	1×14; 2×13; 3×12; 4×11; 5×10; 6×9; 7×8
36	2×18; 3×12; 4×9; 6×6	1×17; 2×16; 3×15; 4×14; 5×13; 6×12; 7×11; 8×10; 9×9

Note: Dimensions are constrained by the size of the board.

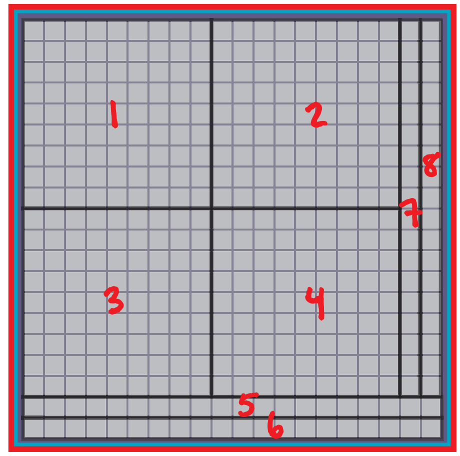
student pairs or groups, before students play the game, so they can have the chart to work with. The chart would be especially helpful for students who struggle with mental math and/or quick calculations.

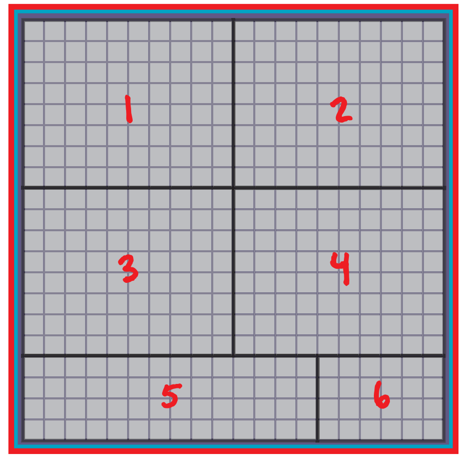
- ***What is the largest possible area that a rectangle in this game can cover?***

  The chart shows that some products used for perimeters can result in areas that are much larger than the products. The largest possible product is 36. Using our chart, we can see that the product 36 allows us to draw a 9 × 9 square, whose area is 81.

- ***What is the minimum number of rectangles required to cover the entire board?***

  This is a little bit of a puzzle for students. Initially, you would think to use as many of the largest-area rectangles you can. The greatest number of 9 × 9 rectangles that can fit on the board is four, no matter where you place them. There are a number of ways to arrange them, but to fill the rest of the board, the fewest additional rectangles you could use would be four, for a total of eight rectangles. See the upper figure to the right. However, there is a way to fill the board with fewer rectangles! If we use four 8 × 10 rectangles instead, and arrange them as shown in the lower figure, then we can fill the remaining space with a 4 × 14 rectangle and a 4 × 6 rectangle, for a total of six rectangles.





Differentiation, Scaling and Extensions

Change the Rules

Make things interesting by allowing students to vary the rules. Rule changes can be as simple or complex as your club decides! Some examples for possible rule changes are:

- Play with just area. This will make the game slightly simpler, and it is a great way to scale down the difficulty.
- You can also play a non-competitive version, in which the goal is for the two players to work together to try to fill the entire board with rectangles. They can work together to place the rectangles and figure out the best dimensions.
- Change how the game is scored—for example, with points based on area or perimeter.
- Allow for other shapes, such as right triangles or L-shapes.

Make a New Board

Since the board is a simple design, it's easy to change, and a few simple changes can make the game more exciting for future rounds. Some ideas for changing the board are:

- Make a new grid. All you need is some graph paper! Make a grid smaller than 20 × 20 for a quicker game. Make the grid larger to let more people play!
- Change the shape of the board. Make it a rectangle that is not a square, or make it any shape that your club members want!
- Block out certain areas on the grid to make obstacles to avoid. Draw different shapes and shade them in to remove them from play.

FENCE ME IN

Scoreboard

Player A

Player B