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Activity Sheet for the February, 2020, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. (a) Expand the product $(x - 1)(x + 1)$.

   (b) Expand the product $(x - 2)(x + 2)$.

   (c) Expand the product $(x - y)(x + y)$.

2. (a) Evaluate $5^2 - 4^2$.

   (b) Evaluate $6^2 - 5^2$.

   (c) Evaluate $7^2 - 6^2$.

   (d) Evaluate $8^2 - 7^2$.

   (e) Do you notice a pattern in your answers to the first four parts? Will the pattern continue? Why or why not?

3. Evaluate $29 \times 31 + 19 \times 21$.

4. The sum of two positive integers is 11, and the positive difference between their squares is 55. What is the positive difference of the two integers?

The Problems

First Problem: If a, b, c and d are positive integers such that $1 - \dfrac{1}{324} = \dfrac{a}{b} \cdot \dfrac{c}{d}$, what is the least possible value of $a + c$?

Second Problem: The circles given by the equations $x^2 + y^2 = 169$ and $x^2 + (y - 14)^2 = 225$ have a common chord. How many units long is that chord?

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Follow-up Problems

5. What is the product of the two smallest prime factors of $2^{1024} - 1$?

6. The difference between the squares of two positive integers is 2009. What is the maximum possible difference between these two integers?

7. Three consecutive positive odd integers a, b and c satisfy $b^2 - a^2 = 344$ and $c^2 - b^2 > 0$. What is the value of $c^2 - b^2$?

8. Great Aunt Minnie's age plus the square of Great Uncle Paul's age is 7308. Great Uncle Paul's age plus the square of Great Aunt Minnie's age is 6974. What is the sum of Great Aunt Minnie's age and Great Uncle Paul's age?

Wow! Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).