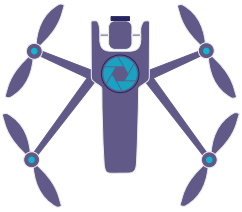
THE NATIONAL MATHCLUB

Exploring STEM Careers

Math is the foundation of STEM. There are many different STEM careers, but all involve applying math in some way to achieve their goals. Take some time to explore just a few of these careers below, by reading a brief overview of what they do and working on two example problems they commonly encounter. Feel free to skip around to figure out which careers might interest you!

Drones, the most common of the unmanned vehicles, are designed, developed and improved by drone engineers. Drones have a wide variety of purposes and functions, so drone engineers must have training in many different engineering fields, such as robotics, aeronautical, mechanical and electrical engineering.



1. An unmanned aerial vehicle (UAV) carries 45 gallons of fuel. It burns fuel at a rate of 3 gph (gallons/hour) and flies at a speed of 160 mph (miles/hour). We want to fly it to an undisclosed location, fly it over the area for 1 hour and then fly it back to its launching point. What is the greatest number of miles away from the UAV's launching point could the undisclosed location be?

2. A robot has a total battery life of 3 hours while carrying 0 pounds. The robot's maximum velocity is 20 feet per minute. For every pound the robot needs to carry, the total battery life of the robot, while traveling at maximum velocity, decreases by 8 minutes. Starting with a fully charged battery, how many feet can the robot travel at its maximum velocity while carrying 10 pounds?

An actuary is a business professional who analyzes the financial impact of randomly occurring events. Actuaries work primarily in the insurance industry and for state and federal government agencies. Actuaries use math and statistics to determine the probability of major events in a geographic area (like an earthquake in Central California). They also use additional information, like the expected magnitude of an earthquake, to help insurance companies set rates.

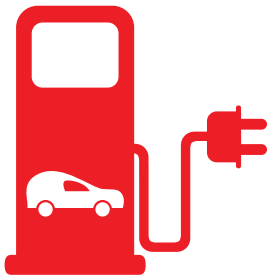
3. The probability that a particular state will have neither a hailstorm nor a tornado in a given month is 55%. In the same period, the probability of a hailstorm is 35% and the probability of a tornado is 25%. If the probability of a hailstorm and the probability of a tornado are not mutually exclusive, what is the probability of both a hailstorm and a tornado occurring in a given month? Express your answer as a percent.

4. A certain disease is expected to infect 1 out of every 10,000 individuals in acountry. A test for the disease is 99.5% accurate. It never gives a false indication when it is negative, so 0.5% of the people who take the test will get inaccurate readings, all of which will be false positives (meaning that the people test positivebut do not have the disease). Let us suppose you test positive; what is the probability that you actually have the disease? Express your answer as a percent to the nearest whole number.

Electrical engineers design, develop and improve electrical systems. They work with all kinds of electrical devices, from small pocket devices to large supercomputers, as well as on the wiring of buildings, telecommunication systems, satellite communication and electrical power systems.

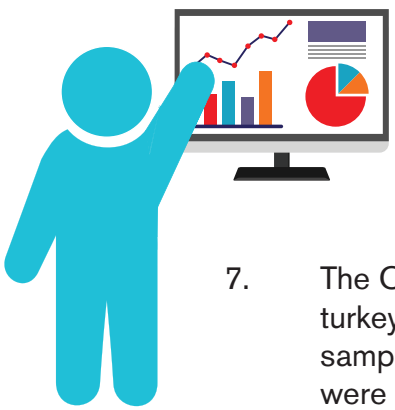
5. An electric circuit in a house goes from the circuit breaker box in the basement to the kitchen. In the kitchen, there are four outlets connected to this circuit. These are the only things in this circuit. The wire in the wall from the basement to the kitchen is able to handle 15 amps of current without becoming too hot. The circuit breaker for this circuit in the basement protects the wiring in the wall from damage. The circuit breaker will trip if more than 15 amps of current flows through the circuit. The house has standard 120-volt electric service. In the kitchen, there is one thing connected to the outlets in the circuit - an 1100-watt toaster. The power required for operating the toaster will be pulled from the total power available to the circuit. The homeowner wants to purchase a coffeemaker and connect it to another outlet in the same circuit. What is the maximum power, in watts, that the coffeemaker can consume without causing the circuit breaker in the basement to trip when both the coffeemaker and the toaster are operating simultaneously? Note: Power (watts) = voltage (volts) × current (amps).

6. Plug-in hybrid electric vehicles (PHEVs) combine one or more electric motors and a gas or diesel engine for propulsion. Energy is stored in a battery pack for the electric motor and in the gas tank for the combustion engine. The combination allows short trips to be made entirely on electric power and raises the effective fuel efficiency of the car. The battery in a typical PHEV stores 9000 watt-hours (W-h) of energy. Some additional information you will need:



   - A house has standard 120-volt electric service.
   - Power = voltage × current (that is, watts = volts × amperes)
   - Energy = power × time (that is, watt-hours = watts × hours)

   A standard wall outlet with a 15-ampere circuit breaker can supply 12 amperes for continuous charging, which avoids tripping the circuit breaker (circuit breakers are set 25% above the maximum continuous current a circuit is designed to carry). Electric vehicle engineers call charging from a standard outlet Level 1 charging. How long will it take to charge a PHEV with a 9000-Wh battery from a standard wall outlet in your home? Express your answer as a decimal to the nearest hundredth.



A statistician gathers, analyzes and interprets data in order to solve problems. They do this by identifying trends and relationships in data and using this information to assist in decision-making and strategy development. This explanation is broad because the scope of what statisticians do is broad as well! A statistician might work in engineering, healthcare, government, private business or many other fields.

7. The Ohio Department of Natural Resources wanted to estimate the number of wild turkeys living in the southwestern part of the state. Zoologists collected an initial sample of 80 turkeys. Identifying bands were attached to these birds, and they were then released into the wild. After two weeks, a second sample of 50 turkeys was selected and 20 were observed to have the identifying bands. What is the estimated number of turkeys in this region of the state?

8. The first polio pandemic hit the United States in 1916 and claimed hundreds of thousands of victims, many of whom were young children. Death could occur in a few short days, and many of those who survived were crippled or paralyzed for life. Dr. Jonas Salk developed a killed-virus vaccine for polio in the early 1950s, and a large clinical trial was needed to see whether the vaccine would protect children against polio. Children whose parents consented to their participation in the study were randomly assigned to receive either the vaccine or the placebo (saline solution) injection. The research study followed the subjects to see whether they contracted polio. The following table reports the results of the Salk Vaccine clinical trial.

Treatment	Polio Incidents	Number of Subjects
Vaccine	57	200,745
Placebo	142	201,229

(i) If medical records of one subject in the clinical trial were randomly selected, what is the probability that this subject contracted polio? Express your answer as a decimal to the nearest ten thousandth.

(ii) Given that the medical records randomly selected is for a subject in the vaccine treatment group, what is the probability that this subject contracted polio? Express your answer as a decimal to the nearest ten thousandth.

Structural engineers contribute to the design of many different types of structures, including bridges, tunnels, office buildings, homes, stadiums and oil rigs. Structural engineers work alongside architects and engineers from other engineering disciplines in the design and construction processes of various structures.

9. A bridge that can support 300 pounds runs across a creek. A man weighing 180 pounds carrying 8.50 gallons of water crosses the bridge. The water weighs 8.33 lb/gal. What is the remaining load bearing capacity of the bridge? Express your answer as a decimal to the nearest tenth.

10. A structural engineer wants to determine the total design wind force on a wall that is 50 feet wide and 70 feet tall. The wind pressure is a constant 15 pounds per square foot (psf) for the lower 15 feet of the wall, then increases linearly to a maximum of 30 psf at the top of the wall. What is the total design force on the wall, in pounds, given that force is the product of applied pressure times the area over which the force is applied?

Club Leader Solution Guide!

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1. An unmanned aerial vehicle (UAV) carries 45 gallons of fuel. It burns fuel at a rate of 3 gph (gallons/hour) and flies at a speed of 160 mph (miles/hour). We want to fly it to an undisclosed location, fly it over the area for 1 hour and then fly it back to its launching point. What is the greatest number of miles away from the UAV's launching point could the undisclosed location be?

   The total time the UAV can travel is (45 gallons)/(3 gallons per hour) = 15 hours. However, one of those hours will be spent flying over the undisclosed location and the UAV has to be able to get back home (that is, it's a round trip), so the UAV can travel up to (15 hours – 1 hour)(160 miles per hour)/2 = 1120 miles from the launching point.

2. A robot has a total battery life of 3 hours while carrying 0 pounds. The robot's maximum velocity is 20 feet per minute. For every pound the robot needs to carry, the total battery life of the robot, while traveling at maximum velocity, decreases by 8 minutes. Starting with a fully charged battery, how many feet can the robot travel at its maximum velocity while carrying 10 pounds?

   While carrying 10 lb the robot will lose (10 lb)(8 min/lb) = 80 minutes of operating time at maximum velocity. Based on the information in the problem, the robot can travel at maximum velocity with no weight for (3 hours)(60 min/1 hour) = 180 min. When the robot loses 80 minutes, that leaves 180 min – 80 min = 100 min in which it can operate carrying 10 lb at maximum velocity. This means the robot will be able to travel up to (100 min)(20 feet/min) = 2000 feet.

3. The probability that a particular state will have neither a hailstorm nor a tornado in a given month is 55%. In the same period, the probability of a hailstorm is 35% and the probability of a tornado is 25%. If the probability of a hailstorm and the probability of a tornado are not mutually exclusive, what is the probability of both a hailstorm and a tornado occurring in a given month? Express your answer as a percent.

   Since the likelihood of neither a hailstorm nor a tornado occurring is 55%, we know that 45% of the time a hailstorm and/or a tornado will occur. The problem states that 35% of the time a hailstorm occurs and 25% of the time a tornado occurs. If these two items were mutually exclusive, 35% + 25% = 60% of the time a hailstorm OR a tornado would occur. However, we know that these two events occur only a total of 45% of the time and we know that these events are NOT mutually exclusive. Since we know that these events (a hailstorm and/or a tornado) occur 45% of the time, we know that 60% – 45% = 15% of the time, these events must occur together.

4. A certain disease is expected to infect 1 out of every 10,000 individuals in acountry. A test for the disease is 99.5% accurate. It never gives a false indication when it is negative, so 0.5% of the people who take the test will get inaccurate readings, all of which will be false positives (meaning that the people test positivebut do not have the disease). Let us suppose you test positive; what is

the probability that you actually have the disease? Express your answer as a percent to the nearest whole number.

Suppose we have 10,000 people. Then, only 1 will have the disease. Now, 0.5% of 10,000 is 50; so 50 people will test positive but not have the disease. So, the total number of positive results is 50 + 1 = 51; thus, the probability that you actually have the disease is 1/51 = 0.019, or 2%.

5. An electric circuit in a house goes from the circuit breaker box in the basement to the kitchen. In the kitchen, there are four outlets connected to this circuit. These are the only things in this circuit. The wire in the wall from the basement to the kitchen is able to handle 15 amps of current without becoming too hot. The circuit breaker for this circuit in the basement protects the wiring in the wall from damage. The circuit breaker will trip if more than 15 amps of current flows through the circuit. The house has standard 120-volt electric service. In the kitchen, there is one thing connected to the outlets in the circuit - an 1100-watt toaster. The power required for operating the toaster will be pulled from the total power available to the circuit. The homeowner wants to purchase a coffeemaker and connect it to another outlet in the same circuit. What is the maximum power, in watts, that the coffeemaker can consume without causing the circuit breaker in the basement to trip when both the coffeemaker and the toaster are operating simultaneously? Note: Power (watts) = voltage (volts) × current (amps).

The circuit breaker will trip if 15 amps of current flow through it. The house has standard 120-volt electric service, so the total power consumed by all of the devices on the circuit together cannot exceed 15 amperes × 120 volts = 1800 watts. The toaster consumes 1100 watts, so the coffeemaker can consume up to 1800 – 1100 = 700 watts of power without tripping the circuit breaker.

6. Plug-in hybrid electric vehicles (PHEVs) combine one or more electric motors and a gas or diesel engine for propulsion. Energy is stored in a battery pack for the electric motor and in the gas tank for the combustion engine. The combination allows short trips to be made entirely on electric power and raises the effective fuel efficiency of the car. The battery in a typical PHEV stores 9000 watt-hours (W-h) of energy. Some additional information you will need:
   - A house has standard 120-volt electric service.
   - Power = voltage × current (that is, watts = volts × amperes)
   - Energy = power × time (that is, watt-hours = watts × hours)

   A standard wall outlet with a 15-ampere circuit breaker can supply 12 amperes for continuous charging, which avoids tripping the circuit breaker (circuit breakers are set 25% above the maximum continuous current a circuit is designed to carry). Electric vehicle engineers call charging from a standard outlet Level 1 charging. How long will it take to charge a PHEV with a 9000-Wh battery from a standard wall outlet in your home? Express your answer as a decimal to the nearest hundredth.

The power available for continuous charging from a standard wall outlet is 120 volts × 12 amperes = 1440 watts. Use the equation energy = power × time to figure out how much time is required to charge the PHEV. The time required is energy divided by power, or for a Level 1 charging outlet, 9000 W-h ÷ 1440 W = 6.25 hours.

7. The Ohio Department of Natural Resources wanted to estimate the number of wild turkeys living in the southwestern part of the state. Zoologists collected an initial sample of 80 turkeys. Identifying

bands were attached to these birds, and they were then released into the wild. After two weeks, a second sample of 50 turkeys was selected and 20 were observed to have the identifying bands. What is the estimated number of turkeys in this region of the state?

If n = total number of turkeys in southwest Ohio, then $80/n$ is the proportion of the population tagged in the first sample. The observed proportion of tagged turkeys in the second sample was 20/50. Equating these proportions leads to an estimate of the population size: $80/n = 20/50$ or estimated $n = 80 \times (50/20) =$ 200 turkeys.

8. The first polio pandemic hit the United States in 1916 and claimed hundreds of thousands of victims, many of whom were young children. Death could occur in a few short days, and many of those who survived were crippled or paralyzed for life. Dr. Jonas Salk developed a killed-virus vaccine for polio in the early 1950s, and a large clinical trial was needed to see whether the vaccine would protect children against polio. Children whose parents consented to their participation in the study were randomly assigned to receive either the vaccine or the placebo (saline solution) injection. The research study followed the subjects to see whether they contracted polio. The following table reports the results of the Salk Vaccine clinical trial.

Treatment	Polio Incidents	Number of Subjects
Vaccine	57	200,745
Placebo	142	201,229

(i) If medical records of one subject in the clinical trial were randomly selected, what is the probability that this subject contracted polio? Express your answer as a decimal to the nearest ten thousandth.

The probability a randomly selected subject from the entire trial contracted polio is equivalent to the total number of incidents over the total number of trial subjects or (57 + 142)/(200,745 + 201,229) = 199/401974 = .000495.

(ii) Given that the medical records randomly selected is for a subject in the vaccine treatment group, what is the probability that this subject contracted polio? Express your answer as a decimal to the nearest ten thousandth.

The probability a randomly selected subject who received a vaccine treatment contracted polio is equivalent to the number of polio incidents of the vaccine group only over the number of subjects who received the vaccine or 57/200,745 = .000283.

9. A bridge that can support 300 pounds runs across a creek. A man weighing 180 pounds carrying 8.50 gallons of water crosses the bridge. The water weighs 8.33 lb/gal. What is the remaining load bearing capacity of the bridge? Express your answer as a decimal to the nearest tenth.

The water weighs 8.50 gal × 8.33 lb/gal = 70.8 lb, to the nearest tenth. So, the total weight of the man and the water he is carrying is 70.8 + 180 = 250.8 lbs. Therefore, the remaining load bearing capacity of the bridge is 300 − 250.8 = 49.2 lbs.

10. A structural engineer wants to determine the total design wind force on a wall that is 50 feet wide and 70 feet tall. The wind pressure is a constant 15 pounds per square foot (psf) for the lower 15

feet of the wall, then increases linearly to a maximum of 30 psf at the top of the wall. What is the total design force on the wall, in pounds, given that force is the product of applied pressure times the area over which the force is applied?

There are several methods to arrive at the correct answer. One method is the summation of 15 psf acting on the lower portion of the wall and the average force acting on the upper portion of the wall. Therefore: (15 psf)(50 ft × 15 ft) + ((15 psf + 30 psf)/2)(50 ft × 55 ft) = 73,125 pounds.