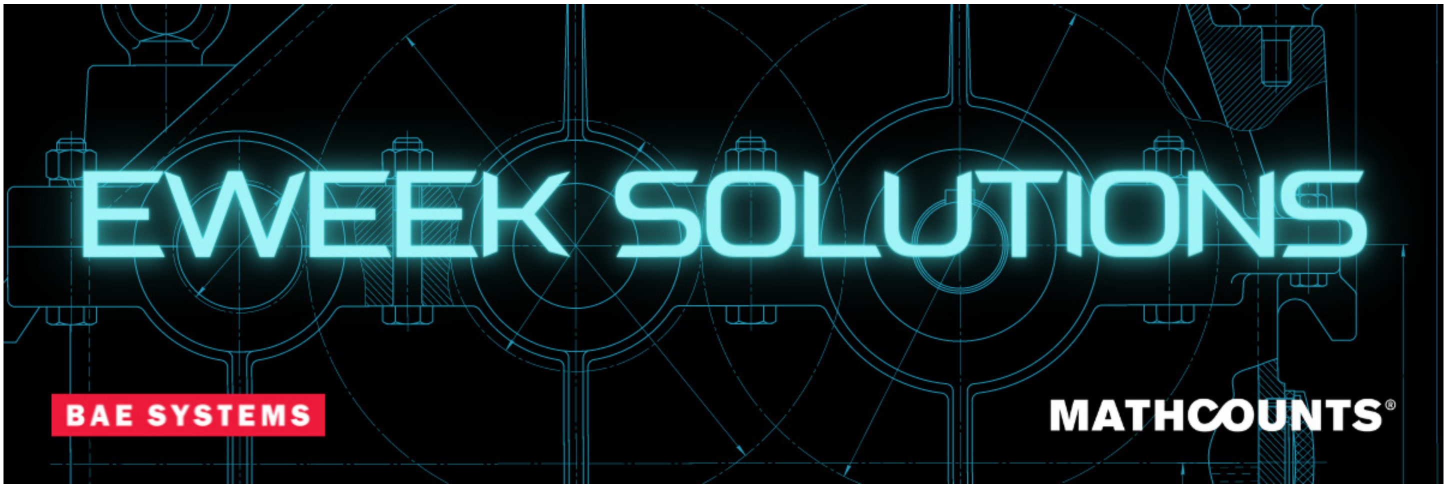


FEB. 21-25 • SOLUTIONS TO E-WEEK PROBLEM SETS

MONDAY, FEB. 21: GEOMATICS ENGINEERING

1.1 In order to determine the height of the monument to Mayor Rockstone, we can use the provided information about the telephone pole and the monument's shadow to set up and solve a proportion, where x is the height of the monument in feet: 35 ft/x = 15 ft/257 ft → $15x = 8995$ → $x \approx$ **600 feet**.

1.2 Substituting 4.03×10^{-7} for t in the formula $d = (t \times 983{,}571{,}056)/2$, we find that the length of the plot is $(4.03 \times 10^{-7})(983{,}571{,}056)/2 \approx$ **198 feet**.

1.3 We are given that the width of the plot is 110 feet. We know that 30% is equivalent to 0.30, so across the width, the land rises 0.30(110) = **33 feet**. Alternatively, since we know the slope of the increase in height is 3/10, we can use the linear equation $y = (3/10)x$ to determine the height y the land has risen over a given distance x across the width of the plot. Consider the lowest point along the width of the plot to be at (0, 0). Then the highest point will be at (110, y). Substituting for x in the linear equation, we see that the land rises to a height of $y = (3/10) \times 110 =$ **33 feet**.

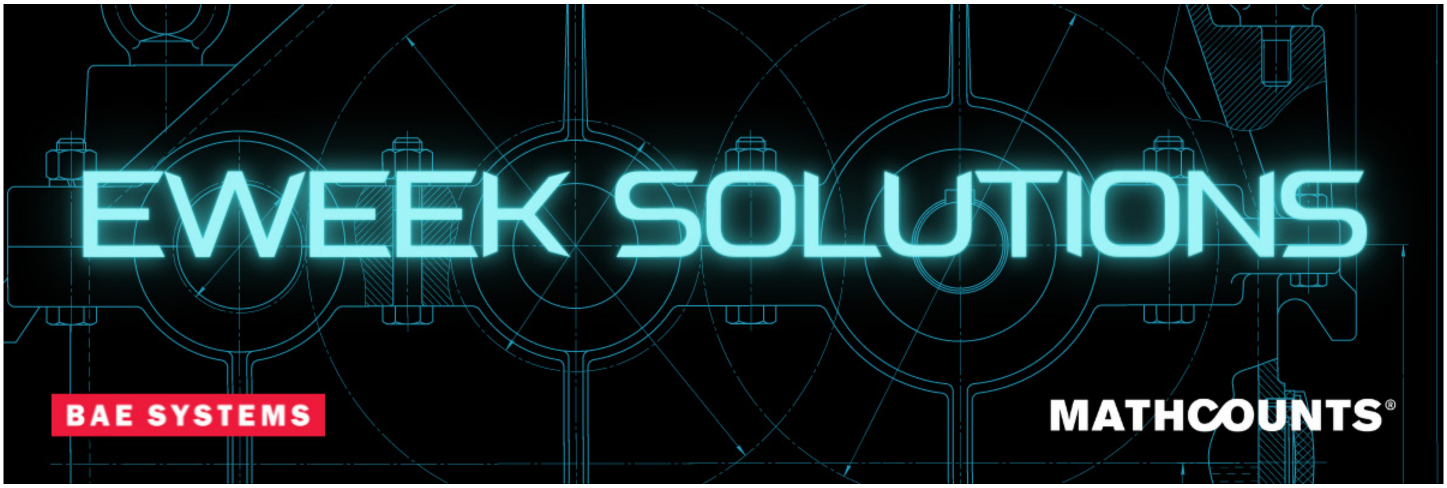
1.4 We know that 1 yd = 3 ft. To find the number of cubic feet in a cubic yard, we can cube each side to get 1 yd^3 = 27 ft^3. So, the amount of dirt that needs to be removed is 135,000/27 = 5000 yd^3. At a cost of $125 per cubic yard, the total cost to remove the necessary amount of dirt to level the plot of land will be 5000 × 125 = **$625,000**.

TUESDAY, FEB. 22: STRUCTURAL ENGINEERING

2.1 The five squares of side length 8 inches in this column's cross-section have a total area of $5 \times 8^2 = 320$ in^2. Since the radius of each quarter-circle is equal to the side length of one of the squares, they have a combined area equal to that of a full circle whose radius is 8 inches, or $\pi \times 8^2 = 64\pi$ in^2. Therefore, the total cross-sectional area of the column is $320 + 64\pi \approx$ **521 in^2**.

2.2 Since all of these loads are acting on the bridge at the same time, the total load is 1,161,080 + 632,500 + 98,000 + 8500 = 1,900,080 pounds. So, we can substitute this total load and the material strength of concrete into the equation load = material strength × area and solve for the cross-sectional area: 1,900,080 = 4200 × area → area = 1,900,080/4200 = **452.4 in^2**.

2.3 The formula for the volume of a right cylinder is $V = \pi r^2 h$. The cross-sectional area will be that of a circle of diameter 3 feet = 36 inches and radius 18 inches. We know that πr^2 is the area of a circle, so the cross-sectional area is $\pi \times 18^2 = 324\pi$ in^2. Since we are told each column has height 14 feet = 14 × 12 = 168 inches, we can substitute into the right cylinder volume formula to get $V = 324\pi \times 168 = 171{,}003.1713202$ in^3 as the volume of each column. Finally, if there will be 8 columns, a total of $171{,}003.1713202 \times 8 \approx$ **1,368,025 in^3** of concrete will be needed to make the columns.



WEDNESDAY, FEB. 23: SYSTEMS & MECHANICAL ENGINEERING

3.1 First, we need to figure out the square footage of the conference room. If this room were rectangular, without the corner cutouts, it would have length 60 feet, width 22 + 28 = 50 feet, and total area 60 × 50 = 3000 ft^2. However, since the room is not rectangular, we must subtract out the areas of the corner cutouts. The corner cutout on the upper right has an area of 22 × 10 = 220 ft^2. The corner cutout on the upper left has an area of 12 × (50 – 34) = 12 × 16 = 192 ft^2. So, the square footage of MATHCOUNTS' conference room is 3000 – (220 + 192) = 3000 – 412 = 2588 ft^2. So, the minimum breathing zone outdoor airflow to accommodate 11 staff members is V_{bz} = (5 × 11) + (0.06 × 2588) = 55 + 158.28 = **210.28 cfm**.

3.2 If we want to increase the air flow rate by 20% and air flow is directly proportional to the fan speed, then we need to increase the fan speed by 20%. This results in a fan speed of 1500 × 1.2 = **1800 rpm**. Alternatively, if we want to increase the air flow rate by 20%, then we want the air flow to be 370 × 1.2 = 444 cfm. We can set up a proportion to determine the fan speed needed to produce this new air flow rate: 1500 rpm/370 cfm = x/444 cfm. Cross multiplying, we get 370x = 666,000 → x = **1800 rpm**.

3.3 We are told that the square of the ratio of the new propeller speed to the old propeller speed is equal to the ratio of the new static pressure to the old static pressure. This can be written as:

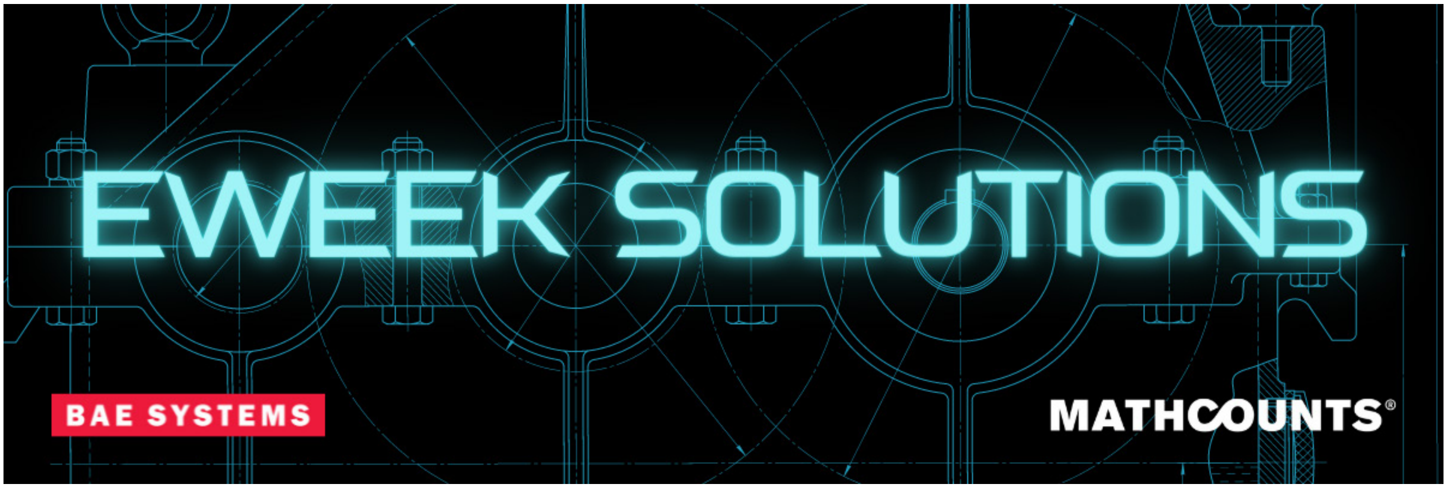
$$\left(\frac{\text{new propeller speed}}{\text{old propeller speed}}\right)^2 = \frac{\text{new static pressure}}{\text{old static pressure}}$$

We know that the old propeller speed is 2000 rpm, and the new propeller speed will be 1850 rpm. We can substitute these values into the equation above to get $(1850/2000)^2 \approx 0.86$. So, (new static pressure)/(old static pressure) ≈ 0.86 as well, which means that the new static pressure is about 86% of the old static pressure. Therefore, the static pressure will decrease by about 100 – 86 = **14%**.

THURSDAY, FEB. 24: NUCLEAR ENGINEERING

4.1 To find the total weight of the uranium fuel, we multiply (10 g per pellet) × (250 pellets per fuel rod) × (264 fuel rods per assembly) × (137 fuel assemblies) = 90,420,000 g. Because we're looking for the total weight of the uranium fuel in kilograms, we must divide by 1,000 grams per kilogram to get 90,420,000 ÷ 1,000 = **90,420 kg**.

4.2 To solve this problem, we must consider the following scenarios: 1 broken fan, 2 broken fans and 3 broken fans. The numbers of combinations of broken fans for these scenarios are ${}_{10}C_1$ = 10!/(1!9!) = 10, ${}_{10}C_2$ = 10!/(2!8!) = 45 and ${}_{10}C_3$ = 10!/(3!7!) = 120, respectively. Therefore, there are a total of ${}_{10}C_1 + {}_{10}C_2 + {}_{10}C_3$ = 10 + 45 + 120 = **175 combinations**.



4.3 If the hospital needs 50 g of ^{99m}Te, this means that at the end of the 48 hours that transpired from the time it was produced until it was delivered, there must be 50 g remaining. We can work backwards from hour 48 to hour 0 (when it was produced) to figure out how much should be shipped. There are 48 ÷ 6 = 8 sets of 6 hours in this time frame. We must double the remaining 50 g, and continue to double the result, for each set of 6 hours (or 8 times). Doing so, we get $50 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 50 \times 2^8 = 12{,}800$ = **1.28×10^4 g**.

FRIDAY, FEB. 25: ELECTRICAL ENGINEERING

5.1 We can substitute the values we know into the formula run time = (10 × battery capacity)/appliance load and solve for battery capacity: 12 = (10 × battery capacity)/2.6 → 31.2 = 10 × battery capacity → battery capacity = 31.2/10 = **3.12 Ah**.

5.2 The battery capacity c for Jared's phone is 3.72 Ah. Substituting this value for c into the equation $t = 2c$, we see that it will take Jared's phone $t = 2(3.72) = 7.44 \approx$ **7.4 hours** to fully charge.

5.3 First, we can determine the battery capacity of Marnie's phone by using the equation $t = 2c$. If the battery takes 4 hours to charge, then we have $4 = 2c$, which means the battery capacity is $c = 4/2 = 2$ Ah. We can then substitute this value into the formula run time = (10 × battery capacity)/appliance load to find the maximum run time of her phone's battery: run time = $(10 \times 2)/2.6 = 20/2.6 \approx$ **7.7 hours**.

GEOMATICS

MONDAY, FEBRUARY 21

BAE SYSTEMS

MATHCOUNTS®

E-WEEK ● GEOMATICS ENGINEERING

Geomatics engineering is all about geographic information. *Geomatics engineers collect this data (such as dimensions of plots of land, details of natural land formations, elevation, angles and boundaries), then analyze and interpret it to find solutions to measurement-related issues and design mapping solutions. They also distribute this information to others who may be building on, using or studying specific land. Geomatics engineering plays an important role in construction, transport, communication, mapping and research.*

1.1 In Rockstone City, no building is permitted to be taller than the monument to the city's first mayor, Mayor Rockstone. A new developer wants to build an apartment building in Rockstone City, but does not know how tall the monument to Mayor Rockstone is. A 35-foot telephone pole next to the monument casts a shadow that is 15 feet long. If the monument's shadow is 257 feet long at the same time of day, how tall is the monument to Mayor Rockstone? Express your answer to the nearest whole number of feet.

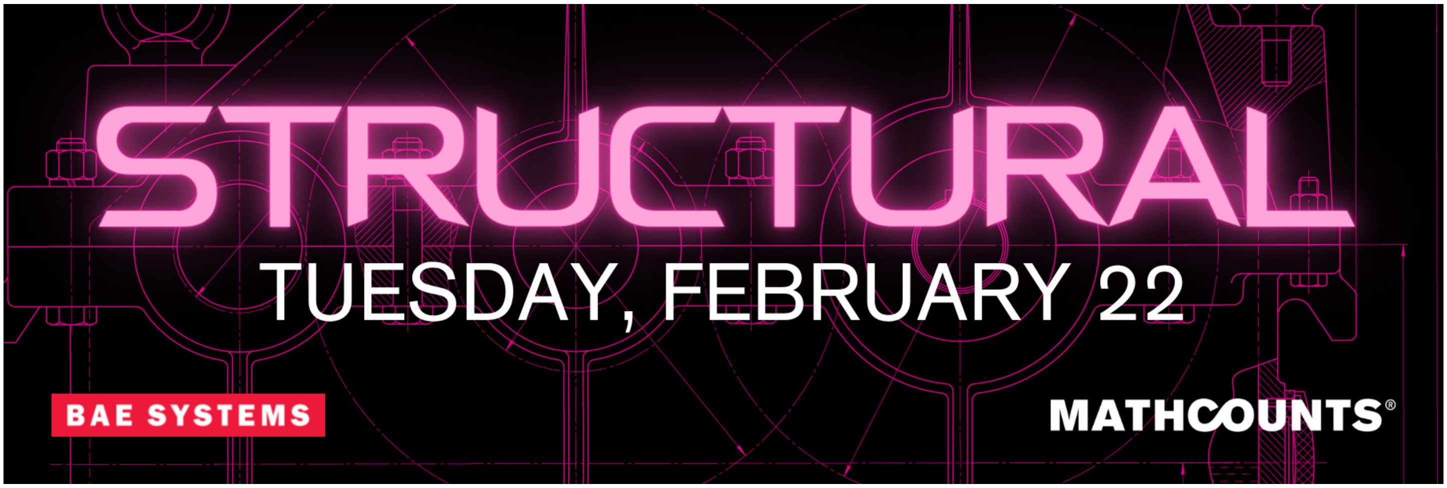
1.2 ***LiDAR***, or **Li**ght **D**etection **a**nd **R**anging, is a method used in geomatics engineering to determine distances and other measurements of natural formations either on land or underwater. To measure distance using LiDAR, a laser light source is shot from one point to another and bounces back. The distance *d*, in feet, between the two points is calculated by multiplying the time *t*, in seconds, it takes the laser light to travel from one point to another and back by the speed of light *c*, in feet per second, and then dividing the result by 2. Since the light travels at a constant speed of 983,571,056 ft/s, we can use the following formula:

$$d = \frac{t \times 983{,}571{,}056}{2}$$

The developer uses LiDAR to determine the length of the rectangular plot of land on which she plans to build the apartment building. If the time it took the laser light to travel the length of the plot and back was 4.03×10^{-7} seconds, how long is the plot of land? Express your answer to the nearest whole number of feet.

1.3 The rectangular plot of land, which has a width of 110 feet, has a ***grade*** (or slope) of 30%, which means over 100 feet, the land rises 30 feet. Assuming the land rises across the width of the plot, how many total feet does the land rise?

1.4 Before beginning construction, the ground must be leveled in order to build on a flat surface. If the company contracted by the developer charges $125 per cubic yard for dirt removal, what will be the total charge to remove 135,000 ft^3 of dirt to level this plot of land?

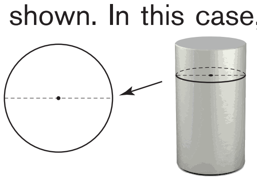


E-WEEK ● STRUCTURAL ENGINEERING

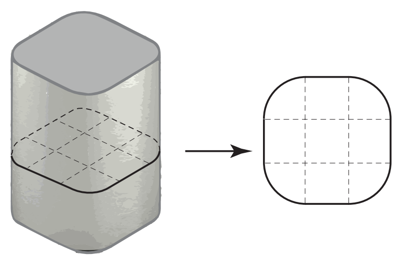
Structural engineering is a sub-field of civil engineering in which structures like bridges, buildings and even things like concert stages are designed and analyzed. *Structural engineers must consider environmental factors, what a structure will be used for and what type of weight will be put on it, and how this all might change from day to day before even beginning a design. Their work keeps us safe in and around all types of man-made structures.*

When designing a bridge, there are two main structural pieces: ***beams*** and ***columns***. The size of the columns and beams is determined by the ***loads*** (or types and amount of weight) that the bridge will support, as well as natural forces that may be applied. For example, a bridge that will support trains and cars will require much larger beams and columns than a bridge that supports pedestrians and bicycles. A bridge in a location that experiences tornadoes will have different specifications than a bridge in a location that experiences heavy snow or earthquakes.

The ***cross-sectional area*** of a column is used to determine the amount of load it can support. A cylindrical column, for example, has a circular cross-section as shown. In this case, the cross-sectional area is just the area of the circle.



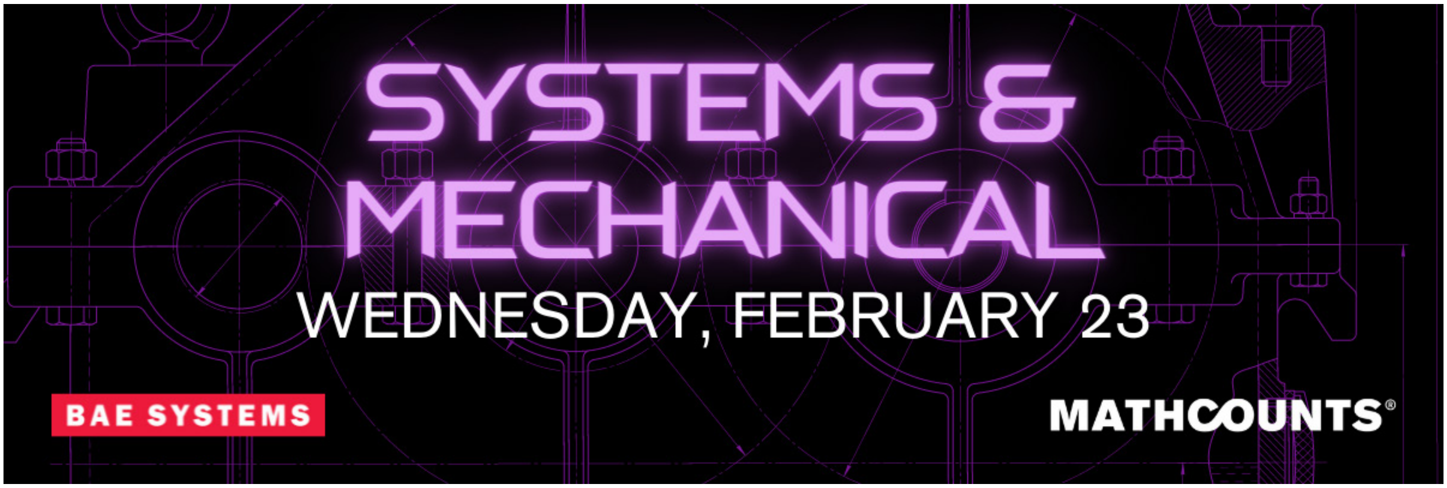
2.1 The figure shows the cross-section of a bridge column. As the dashed lines show, this cross-section is composed of 5 congruent squares of side length 8 inches and 4 congruent quarter circles. What is the cross-sectional area of this bridge column? Express your answer to the nearest whole number.



2.2 The formula shown is used to determine the ***maximum load***, in pounds, that a column can support. The area refers to the cross-sectional area of the column, and material strength is a constant value that is assigned to each type of building material. An engineer is designing a concrete bridge across a river in Michigan, where they must account for weather conditions. For this particular bridge, the ***dead load*** (weight of fixed elements, like the concrete roadway and signs) is 1,161,080 pounds; the ***live load*** (weight of temporary elements, like vehicles and people) is 632,500 pounds; the ***snow load*** (temporary weight from snow) is 98,000 pounds; and the ***wind load*** (temporary weight from wind) is 8500 pounds. If it were to be snowing and windy at the busiest time of day on the bridge, all of these loads would be acting on the bridge at the same time. Given that the material strength of concrete is 4200 lb/in^2, what would the cross-sectional area of the column need to be to support this load? Express your answer as a decimal to the nearest tenth.

$$\text{load (lbs)} = \text{material strength (lbs/in}^2\text{)} \times \text{area (in}^2\text{)}$$

2.3 This bridge will have 8 identical right circular cylinder columns, each with diameter 3 feet and height 14 feet. Based on this, what is the total volume, in cubic inches, of the concrete needed to make these columns? Express your answer to the nearest whole number.



E-WEEK ● SYSTEMS & MECHANICAL ENGINEERING

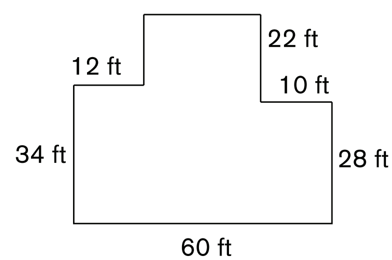
Systems engineers work in many different engineering fields. *They often serve as both project manager and chief engineer. In mechanical engineering in particular, systems engineers need to ensure individual sub-systems and components of a large project all work together to perform the overall desired function. Many professionals in mechanical engineering work in* ***HVAC*** *(heating, ventilation and air conditioning). These systems are essential to keeping indoor spaces safe and comfortable.*

An ***outdoor air ventilation system*** is used to regulate the ***breathing zone outdoor airflow*** (V_{bz}) in a space. In other words, it ensures that the amount of outdoor air, measured in cubic feet per minute (cfm), in a space meets a particular standard based on the function of the space. These standards are set by the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE). As the formula shows, HVAC engineers must consider the square footage and the number of people occupying a space when determining the total amount of outdoor air needed.

$$V_{bz} = (R_p \times P_z) + (R_a \times A_z)$$

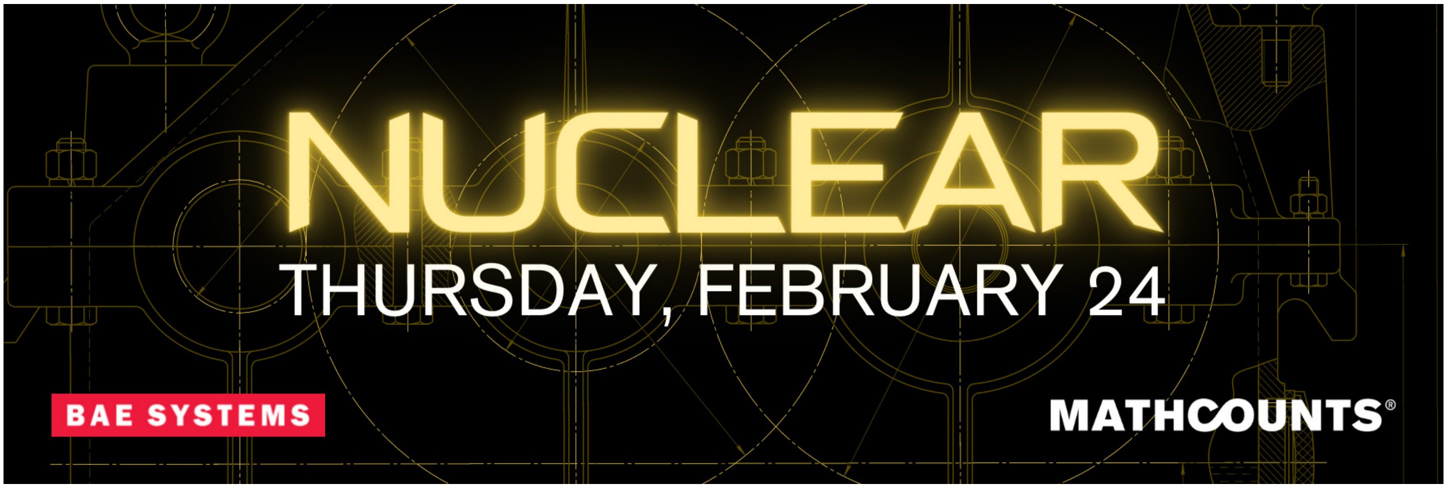
- R_p: outdoor airflow rate per person (cfm)
- P_z: zone population (# people during typical usage)
- R_a: outdoor airflow rate per unit area (cfm)
- A_z: zone floor area (ft^2)

3.1 Suppose MATHCOUNTS hires an HVAC systems engineer to install an outdoor air ventilation system in our office building. The figure shows the dimensions of the MATHCOUNTS conference room. According to ASHRAE standards for office buildings, R_p = 5 cfm and R_a = 0.06 cfm. Based on this, what V_{bz} should the ventilation system maintain in order for this conference room to accommodate the MATHCOUNTS staff of 11? Express your answer as a decimal to the nearest hundredth.



3.2 Ventilation plays a key role in air conditioning systems. By using fans, the systems prevent stagnation and ensure that air continues to circulate. In MATHCOUNTS' system, the fans are currently operating at a speed of 1500 revolutions per minute (rpm), producing an air flow rate of 370 cfm. This rate, however, is determined to be about 20% below the acceptable rate for air circulation. Given that the change in speed of the fan's propeller is directly proportional to the change in air flow rate of a fan, at what speed, in revolutions per minute, should the fans in the MATHCOUNTS' system operate for the air to flow at a rate that is 20% greater than the current air flow rate?

3.3 Later, an HVAC systems engineer who examines the full system notes that the fan is set to operate at 2000 rpm but determines that the ***static pressure*** (the amount of pressure needed by a fan to push and pull air through ductwork) needs to be reduced by lowering the fan speed to 1850 rpm. If the square of the ratio of the new propeller speed to the old propeller speed is equal to the ratio of the new static pressure to the old static pressure, by what percent will the static pressure need to be reduced? Express your answer to the nearest whole percent.

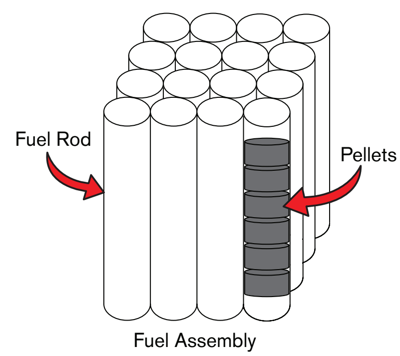


E-WEEK ● NUCLEAR ENGINEERING

Nuclear engineering is all about harnessing the energy that is released by reactions within atoms. *This energy can be used for a variety of things, including generating electricity, powering transportation systems, and diagnosing and treating illnesses. One of the most common places to find nuclear engineers at work is in a nuclear power plant. While nuclear engineers are central to the design, construction and maintenance of power plants, they are also responsible for proper safety procedures and disposal of nuclear waste. Nuclear engineers can also be found in other fields, where they might develop radioactive medicines such as radiation therapy that treats cancers, or design power sources for space satellites or equipment on the moon, etc.*

At the heart of a nuclear power plant is a ***nuclear reactor***, which contains and controls the reactions that occur within the atoms. These reactions produce heat, which is used to boil water to make steam. The steam is then used to turn a turbine and generate electricity.

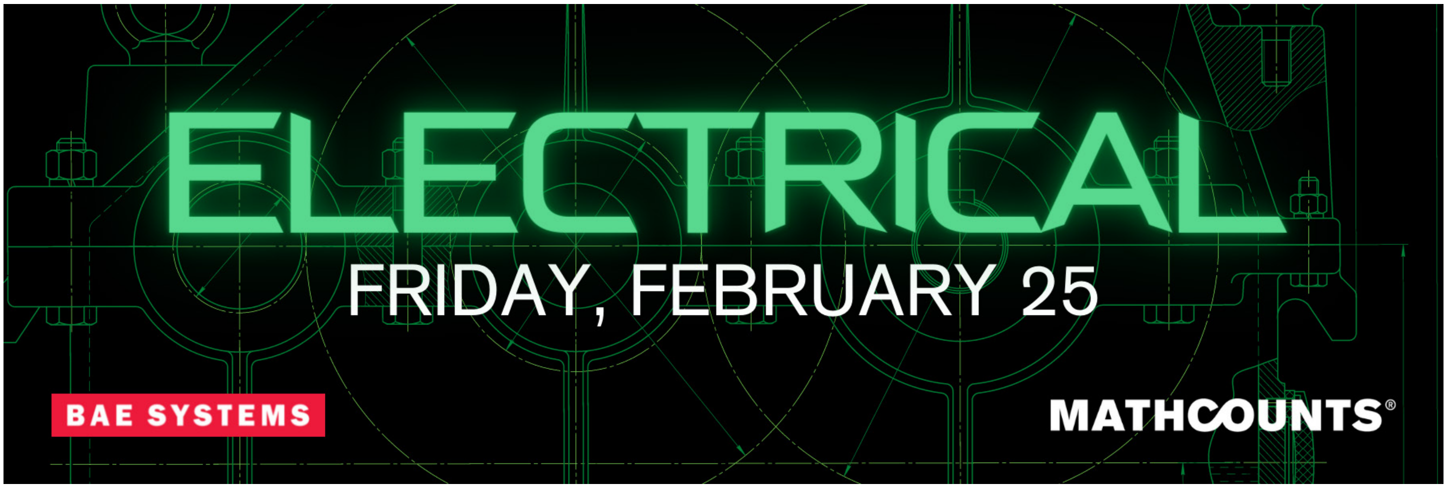
4.1 Nuclear reactors are powered by uranium fuel, which consists of small pellets packaged into long, metal tubes called ***fuel rods***. These fuel rods are then organized into ***fuel assemblies***, each of which is made up of 264 fuel rods. Assume 137 fuel assemblies are needed to fuel a nuclear reactor. If each pellet weighs 10 g, and there are 250 pellets in each fuel rod, what is the total weight, *in kilograms*, of the uranium fuel inside a fully loaded reactor?



4.2 Nuclear power plants produce radioactive waste, which can be dangerous for the environment if it is not disposed of correctly. Often radioactive waste from nuclear power plants is disposed of underground in special storage facilities. In these storage facilities, a high-powered ***HVAC*** (heating, ventilation and air conditioning) system is required to keep the air pressure inside the facility lower than the air pressure outside the facility. This ensures that the radioactive material remains inside the facility and does not flow out into the environment.

The HVAC system in a particular nuclear waste storage facility has 10 fans. There must be 7 or more of these fans running to maintain the low air pressure in the facility. How many different combinations of one or more fans can be broken at the same time *without* causing radioactive material to escape the storage facility?

4.3 Technetium-99m, or ^{99m}Te, is an ***isotope*** (a radioactive form of an element) that is used in nuclear medicine to determine how certain parts of a person's body, particularly their bones, heart muscle or brain, are functioning. The ***half-life*** of ^{99m}Te (or rather, the amount of time it takes for a quantity of ^{99m}Te to decay and be reduced by half) is six hours. If a hospital needs to receive 50 g of ^{99m}Te for its medical procedures, and it takes exactly 48 hours from its production to reach the hospital, how many grams of ^{99m}Te must be shipped to the hospital? Express your answer in scientific notation to three significant figures.



E-WEEK ● ELECTRICAL ENGINEERING

Electrical engineers work on all kinds of projects in many different fields. *Some work on large-scale projects, like urban power grids or communication systems, while others work on tiny microchips that are central to computers and cars. Electrical engineers work on some of our favorite entertainment, like video games and rollercoasters, as well as on some of the most important, life-saving technology, like medical tests, brain implants and prosthetics. One of the common ways we see their work is in our smartphones. Electrical engineers are behind the electronics used in Face ID authentication systems, Wi-Fi connectivity and batteries, among many other things.*

You may have noticed that your microwave at home is an 800-watt microwave. This refers to the ***appliance load***, or the amount of power the appliance uses when in use, and it is often measured in watts (W). ***Battery capacity***, which can be measured in amp hours (Ah), refers to the amount of electric current, or energy, a battery can store and use to supply power to a device for a period of time. The maximum run time, in hours, of a battery can be calculated using the formula

$$\text{run time } (\textit{hours}) = \frac{10 \times \text{battery capacity } (\textit{Ah})}{\text{appliance load } (\textit{W})}$$

5.1 A team of electrical engineers wants a new smart phone's battery to last for 12 hours. If the smart phone has an average appliance load of 2.6 watts, what does the battery capacity need to be, in amp hours? Express your answer as a decimal to the nearest hundredth.

5.2 The charging time t, in hours, of a phone with battery capacity c, in amp hours, can be calculated using the equation $t = 2c$. If Jared's new phone has a battery capacity of 3.72 Ah, how many hours will it take his phone to fully charge? Express your answer as a decimal to the nearest tenth.

5.3 Marnie buys a smart phone from a different company. She heard that this phone takes only 4 hours to charge. Assuming Marnie's smart phone has an average appliance load of 2.6 watts, how many hours can Marnie expect to use her phone before the battery dies? Express your answer as a decimal to the nearest tenth.