THE NATIONAL MATHCLUB

Dictionary Day

1. The letters of the alphabet are each assigned a random integer value, and H = 10. The value of a word comes from the sum of its letters' values. If MATH is 35 points, TEAM is 42 points and MEET is 38 points, what is the point value of A?

2. The "word-sum" of a word is determined by adding together the value of each letter. In the alphabet, letters A through H each have a value of 5 cents; letters I through R each have a value of 7 cents; and letters S through Z each have a value of 8 cents. What is the word-sum of the word MATHCOUNTS, in cents?

3. How many letters of the alphabet shown below have a vertical line of symmetry?

ABCDEFGHIJKLMNOPQRSTUVWXYZ

4. A set of magnetic letters contains two of each consonant and three of each vowel. Only a complete set of letters may be purchased. How many complete sets of magnetic letters must be purchased to make a sign that reads: "MATHCOUNTS COMPETITION TODAY"?

5. The sequence below is formed by repeating the letters of the alphabet, in order, the same number of times as the letter's ordinal position in the alphabet. After 26 Zs, the sequence starts over with ABBCCC.... What is the 2005th letter in the sequence? A B B C C C D D D D E E E E E...

6. If all of the letters of the word BEEP are used, in how many different ways can the four letters be arranged in a four-letter sequence? The two Es are indistinguishable, so EEPB should be counted only once since we would not be able to tell a difference if the two Es were swapped.

(For #7, #8) A value is assigned to each letter of the alphabet such that A = 1, B = 2, C = 3, ..., Z = 26. The "word-product" of a word is the product of the values of each letter in the word. For example, the word-product of NAME is 14 × 1 × 13 × 5 = 910.

7. What is Carrie's favorite breakfast food if its word-product is 4655?

8. What is her favorite dessert if its word-product is 165?

9. A value is assigned to each letter of the alphabet such that A = 1, B = 2, C = 3, ..., Z = 26. A nine-digit code is then created for each letter using the prime factorization of its assigned value. The first digit of a letter's code is the number of times 2 is used as a factor, the second digit is the number of times 3 is used as a factor, the third digit is the number of times 5 is used as a factor and so on. For example, since N is the 14^{th} letter of the alphabet and $N = 14 = 2^1 \times 7^1$, the code for the letter N is 100100000 with 1s in the ***first*** and ***fourth*** positions because its prime factorization has one 2 (the ***first*** prime number) and one 7 (the ***fourth*** prime number). What 6-letter word does the following sequence of six codes represent? The first row is the code for the first letter of the word, the second row is the code for the second letter of the word and so on.

0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0
0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0

10. There are many words that can be made from the 10 letters in the word "DICTIONARY." According to the definition of "word-product" given for #7 and #8, what word can you make with the highest word-product?

Club Leader Solution Guide!

THE NATIONAL MATHCLUB

Dictionary Day

1. The letters of the alphabet are each assigned a random integer value, and H = 10. The value of a word comes from the sum of its letters' values. If MATH is 35 points, TEAM is 42 points and MEET is 38 points, what is the point value of A?

If MATH = 35 points and H = 10 points, then MATH − H = MAT = 35 − 10 = 25. TEAM contains the letters MAT as well, so knowing that TEAM = 42 points, then TEAM − MAT = E = 42 − 25 = 17. Then, since MEET = 38, and E = 17, MEET = M + 17 + 17 + T = M + 34 + T = 38. Using the inverse operation, find that MT = 38 − 34 = 4. Recall that MAT = 25, so MAT − MT = A = 25 − 4 = 21 points.

2. The "word-sum" of a word is determined by adding together the value of each letter. In the alphabet, letters A through H each have a value of 5 cents; letters I through R each have a value of 7 cents; and letters S through Z each have a value of 8 cents. What is the word-sum of the word MATHCOUNTS, in cents?

The letters M (7 cents), A (5 cents), T (8 cents), H (5 cents), C (5 cents), O (7 cents), U (8 cents), N (7 cents), T (8 cents) and S (8 cents) added together equal 7 + 5 + 8 + 5 + 5 + 7 + 8 + 7 + 8 + 8 = 68 cents.

3. How many letters of the alphabet shown below have a vertical line of symmetry?

ABCDEFGHIJKLMNOPQRSTUVWXYZ

11 letters have a vertical line of symmetry. Those letters are A, H, I, M, O, T, U, V, W, X and Y.

4. A set of magnetic letters contains two of each consonant and three of each vowel. Only a complete set of letters may be purchased. How many complete sets of magnetic letters must be purchased to make a sign that reads: "MATHCOUNTS COMPETITION TODAY"?

The most common vowel is O—there are 4 of them. Because there are three of each vowel in a set, this would require 2 sets. However, the most common consonant is T—there are 5 of them. Because there are two of each consonant in a set, this would require 3 sets. Since only complete sets can be purchased, 3 sets must be purchased.

5. The sequence below is formed by repeating the letters of the alphabet, in order, the same number of times as the letter's ordinal position in the alphabet. After 26 Zs, the sequence starts over with ABBCCC.... What is the 2005th letter in the sequence? A B B C C C D D D D E E E E E...

One repetition of this sequence (ABBCCCDDDDEEEEE...) is 351 letters. 351 × 5 = 1755. This is as close as possible to 2005 without going over. So, 2005 − 1755 = 250. The 2005th letter in the whole sequence is the 250th letter in one repetition of this sequence. 250 falls within the part of the sequence where the 22nd letter of the alphabet repeats, which is V.

6. If all of the letters of the word BEEP are used, in how many different ways can the four letters be arranged in a four-letter sequence? The two Es are indistinguishable, so EEPB should be counted only once since we would not be able to tell a difference if the two Es were swapped.

There are 12 ways to arrange these letters, assuming the two Es are indistinguishable: BEEP, PEEB, EPEB, EEPB, EBEP, EEBP, PBEE, BPEE, PEBE, BEPE, EPBE and EBPE.

(For #7, #8) A value is assigned to each letter of the alphabet such that A = 1, B = 2, C = 3, ..., Z = 26. The "word-product" of a word is the product of the values of each letter in the word. For example, the word-product of NAME is 14 × 1 × 13 × 5 = 910.

7. What is Carrie's favorite breakfast food if its word-product is 4655?

Create a prime factor tree and find that 4655 = 5 × 7 × 7 × 19. E = 5, G = 7 and S = 19. So, Carrie's favorite breakfast food is EGGS.

8. What is her favorite dessert if its word-product is 165?

Create a prime factor tree and find that 165 = 3 × 1 × 11 × 5. C = 3, A = 1, K = 11 and E = 5. So, Carrie's favorite dessert is CAKE.

9. A value is assigned to each letter of the alphabet such that A = 1, B = 2, C = 3, ..., Z = 26. A nine-digit code is then created for each letter using the prime factorization of its assigned value. The first digit of a letter's code is the number of times 2 is used as a factor, the second digit is the number of times 3 is used as a factor, the third digit is the number of times 5 is used as a factor and so on. For example, since N is the 14th letter of the alphabet and N = 14 = $2^1 \times 7^1$, the code for the letter N is 100100000 with 1s in the ***first*** and ***fourth*** positions because its prime factorization has one 2 (the ***first*** prime number) and one 7 (the ***fourth*** prime number). What 6-letter word does the following sequence of six codes represent? The first row is the code for the first letter of the word, the second row is the code for the second letter of the word and so on.

In row 1, it says the prime factorization of the number has one 5 (the third prime number). So, the first letter is E, as 5 is the value assigned to E. In row 2, it says the prime factorization of the number has one 17 (the seventh prime number). So, the second letter is Q, as 17 is the value assigned to Q. In row 3, it says the prime factorization of the number has one 3 and one 7 (the second and fourth prime numbers, respectively). So, the third letter is U, as 3 × 7 = 21 is the value assigned to U. In row 4, there is no prime factorization possible, which means the value is 1. A = 1, so the fourth letter is A. In row 5, it says the prime factorization of the number has two 2s and one 3 (the first and second prime numbers, respectively). So, the fifth letter is L, as 2 × 2 × 3 = 12 is the value assigned to L. Finally, in row 6, it says the prime factorization of the number has one 19 (the eighth prime number). So, the sixth letter is S, as 19 is the value assigned to S. The sequence of codes spells EQUALS.

10. There are many words that can be made from the 10 letters in the word "DICTIONARY." According to the definition of "word-product" given for #7 and #8, what word can you make with the highest word-product?

There are multiple answers for this problem—the word TRY has a word-product of 20 × 18 × 25 = 9000, so incorporating these three letters will give multiples of 9000.