DICE DUEL

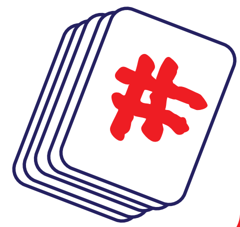
Everything You Need to Play

MATERIALS

Six-Sided Dice

- 2 six-sided dice per pair of students. These instructions will assume students are playing one-on-one, but students can also play team versus team.
- The MATHCOUNTS Club App is available in lieu of physical dice. If the app is used, one device is needed per pair of students.

Product Cards



- 18 product cards per pair of students. In this document you will find a template for these cards. Make copies and cut each copy along the dotted lines. Or use scratch paper and cut or tear each sheet of paper into 18 pieces.

RULES

In Dice Duel, players compete one-on-one to be the first to have all of their selected products rolled. The students will each select 9 of the 18 possible products of two dice and take turns rolling the dice until all of one student's products have been rolled.

- Lay out 18 product cards (or scraps of paper), each containing a different one of the following numbers: 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36.
- Decide who will be Player A and who will be Player B—rock paper scissors, flip a coin, etc.
- To start, the players "draft" products. Player A chooses a product from the 18 available and puts the product in front of him or her. Then Player B chooses a product from the remaining 17. The two players continue to alternate "drafting" products until all 18 are gone and each player has 9 in front of him or her.
- Next, Player A rolls both dice. The players should calculate the product of the two numbers rolled. Whichever player has this product turns over the card with this product on it, and it is no longer in play.
- Beginning with Player B's first turn, the rules change slightly. From then on, the roller has the choice of rolling one die (and leaving the other die showing the number previously rolled) or rolling both dice. Player B should pick up either one die or both dice and then roll. Again, the product is determined either by multiplying the one die rolled and the one die not rolled or by multiplying both dice that were rolled. If the product rolled is displayed on a card in front of either player, he or she should turn over that product card.
- Play continues in this manner, with players taking turns rolling one or both dice and turning over displayed product cards. The winner is the first player to turn over all nine of his or her product cards.

Differentiation, Scaling & Extensions

Change the Rules

Rule variations are easy ways to make the game simpler or harder, depending on the needs of your club. If you feel your members need an easier version of the game or if you feel they are looking for more of a challenge, try the rule changes below. These are only two examples of changes. Have students think of their own rule changes and how they will affect the game!

- Try using sums instead of products. There are 11 possible dice sums (2 through 12). There should be one unclaimed number in the "draft," and each student should have 5 numbers. The calculations are simpler while playing with sums. (*Note*: You will need to create sum cards.)
- Add a third die and play with three players. Then there are 40 possible products, and each player should "draft" 13 numbers. Both the product and probability calculations for this version will take more time.

Extension Idea

Besides the math lesson presented here, there is so much more to dive into with this game. The rules are simple, but the math is endless!

- Talk about the difference between experimental and theoretical probability. Have the students record the results every time they roll the dice. Is each number rolled with the same frequency? Are some products rolled more often than others? Do the experimental probabilities match the theoretical probabilities?
- Make comparisons to the Multiplication Game (available at **www.mathcounts.org/clubleaders**). Although Dice Duel has the added element of probability due to the rolling of the dice, a lot of the concepts of factor pairs and products are applicable here. See if students can make connections between concepts and strategies for the two games.
- Add in products that aren't possible with the factors provided. They will create a dead zone or blocked point on the board that players have to identify.

PRODUCT CARDS

1	2	3
4	5	6
8	9	10
12	15	16
18	20	24
25	30	36

DICE DUEL

Mathematical Exploration

Dice-Rolling Probability

At first glance, this game may appear to students to be more about chance than strategy or skill. But after playing a few rounds, students may begin to observe trends in the numbers being rolled. Have them analyze the probability element of this game. After the lesson, have the students play again and see how the game has changed for them.

- ***After you played the game a few times, were there any products that you were consistently having trouble rolling?***

  Students may or may not have made this observation, depending on how their games went. If you ask multiple groups which products they had trouble rolling, you may notice that certain products are mentioned more often than others. Some products you should expect to see in this list are 1, 9, 16, 25 and 36.

- ***If you noticed some products were more difficult to roll than others, why do you think this is the case? If you didn't make this observation, do you think any products should be easier or harder to roll than others?***

  In fact, not all products are equally probable when rolling dice. There are 36 possible rolls, but 18 possible products. For example, a product of 1 can only be obtained by rolling double 1s, but a product of 6 can be obtained by rolling a 1 and a 6 or a 2 and a 3.

- ***What is the probability of rolling each of the 18 products when BOTH dice are rolled?***

  From least probable to most probable, the probabilities of the products are as follows:

  - 1, 9, 16, 25, 36 = 1/36
  - 2, 3, 5, 8, 10, 15, 18, 20, 24, 30 = 2/36 = 1/18
  - 4 = 3/36 = 1/12
  - 6, 12 = 4/36 = 1/9

x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

  Students can create the chart shown here to make calculating these probabilities easier. The chart shows the results of the 36 outcomes when rolling a pair of dice. To calculate the probability of any product as a fraction, students can count the number of occurrences of that product and put this number over the denominator 36. They can also calculate each probability if they know that any number on one die has a 1/6 probability of being rolled.

- ***If you already have one factor rolled, what is the probability of rolling the other die and obtaining the product you need?***

  For any specific product, if you already have one factor rolled, the probability of rolling the second factor and completing the product will always be 1/6—a higher probability than rolling any product with two dice. For example, if one die shows 2 and you are rolling the other die, the probability of rolling a product of 10 will be 1/6 because you must roll a 5 and there is only one 5 of the six numbers on the die. The probability calculations will get more complex when we begin considering specific scenarios in the game, but understanding how to calculate the probabilities of specific products when rolling one die or two is key to mastering this game.

- ***Here is one game scenario. You have only the following product cards displayed: 15, 16, 20 and 36. It is your turn to roll, and the dice currently on the table show a 5 and a 6. You can choose to re-roll both, re-roll only the 5 die or re-roll only the 6 die. Which choice has the highest probability of resulting in one of the products you need?***

  If you roll both dice, then we can add up the individual probabilities of rolling those products. Rolling a 16 has a probability of 1/36, rolling a 15 has a probability of 2/36, rolling a 20 has a probability of 2/36 and rolling a 36 has a probability of 1/36. So the total probability is (1 + 2 + 2 + 1)/36 = 6/36 = 1/6. If you leave the 6 and roll the other die, then the only desired product you can roll is the 36, and the probability of rolling a 6 and completing the factor pair is 1/6. The last choice you have is to leave the 5 and roll the other die. Then you can either roll a 3 to get the product 15 or roll a 4 to get the product 20. Each of these rolls has a 1/6 probability, so the overall probability is (1 + 1)/6 = 2/6 = 1/3. The best move would be to leave the 5 and roll the other die.

- ***With the same scenario as above, how would your strategy change if your opponent still had the following product cards: 5, 10 and 25?***

  With the added knowledge of your opponent's hand, you probably should not make the same choice for your roll. If you leave the 5 and roll the other die, then you have a (1 + 1 + 1)/6 = 3/6 = 1/2 probability of rolling one of your opponent's products. This means it's more likely you will roll one of your opponent's products rather than one of your own. However, if you leave the 6 and roll the other die, there is a 0 probability of rolling one of your opponent's products since none of his or her products has 6 as a factor. That would make leaving the die that shows 6 the safer move.