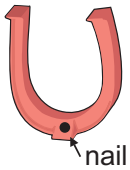
THE NATIONAL MATHCLUB

Defy Superstition Day

1. A horseshoe is thought to hold good luck when it is mounted with the opening facing straight up, as shown. This is how Marge hung the horseshoe on her front door using one nail. However, Kenneth then rotated the horseshoe 135 degrees clockwise on the nail. After that, Abby came along and rotated the horseshoe 240 degrees counterclockwise on the nail. In order to get the horseshoe back to its original position, what is the fewest number of degrees *counterclockwise* Marge would have to rotate the horseshoe?



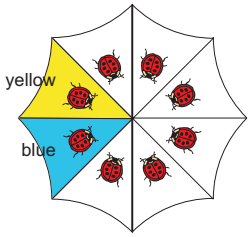
2. Cloie was looking for four-leaf clovers. She was able to find three of them in a very small square patch of clover measuring 18 inches on each side. Based on this patch, how many four-leaf clovers would she expect to find in her backyard of clover that measures 30 feet by 45 feet?

3. Braxton was born on Friday, Feb. 13, 2004. The year 2004 was a leap year. Though his birthday will always be on the 13th, it won't always be on a Friday the 13th or during a leap year. In what year after 2004 will Braxton's birthday first occur on a Friday the 13th?

4. Alexis was hesitant to walk under a ladder, but then she saw a lucky rabbit's foot on the first rung of the ladder, a lucky penny on the fourth rung, and a lucky four-leaf clover on the fifth rung. The rungs are all equally spaced from each other, and the ladder is on level ground. If the rabbit's foot would fall 8 inches straight to the ground and the penny would fall 32 inches straight to the ground, how far would the four-leaf clover fall straight to the ground, in inches?

5. Angel knew that blowing out all five of the candles on her birthday cake in one breath would bring her good luck and would ensure that her birthday wish came true. Unfortunately, three of Angel's five candles were still lit after she tried to blow them all out with her first breath. How many different sets of three candles could have been left burning after her first attempt?

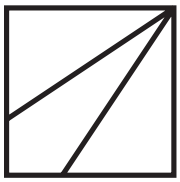
6. Matlyn was so excited about her new Good Luck Ladybug Umbrella that she completely forgot it was bad luck to open the umbrella inside the house. She sprung it open to show her friend Izzy that not only did her umbrella have a lucky ladybug on each of the 8 equally-sized panels, it also had color-symmetry across the vertical, dark black line when looking at the top view shown. If we know the color of the two panels labeled and each panel of the umbrella is either yellow or blue, how many different ways could the remaining panels be colored?



7. Carl saw a penny on the sidewalk that was "heads up," so he knew it would be good luck to pick it up. Then he saw that the year on it was a multiple of 13, so he put the unlucky penny back down. After a couple of seconds, though, he realized the year was also a multiple of lucky 7, so it became lucky again and he picked it back up. If the year on the penny was in the 20th century, what year was on the penny?

8. Martin was trying to avoid the black cat he saw walking up ahead, but was distracted by also trying to avoid stepping on any cracks in the sidewalk. As a result, Martin and the black cat actually walked right into each other! The cat had been 15 feet from the point of the collision when Martin was 20 feet from it. Before the collision the cat had been walking due east at a constant rate of 9 mph. If Martin had been walking due north at a constant rate, how fast was Martin walking, in mph, to cause the collision?

9. Kendra had a square mirror measuring 9 inches on each side. Unfortunately, she dropped the mirror, and it broke into three pieces, as shown, each of equal area. (Don't worry, if the pieces are all buried in the ground under moonlight, she can avoid her 7 years of bad luck.) What is the length of the perimeter, in inches, of the four-sided piece? Express your answer as a decimal to the nearest tenth.



10. The Cougars had only three hits in 24 at-bats during the first eight innings of the game. For the ninth inning, Aaron and his dad put on their rally caps in hopes that the Cougars would come back from a two-run deficit. (Rally caps are baseball caps worn in a silly way, perhaps inside out, to bring good luck to a team.) If the Cougars' first six at-bats in the ninth inning each resulted in hits, and this then ended the game, what percent of the Cougars' at-bats during the entire game resulted in hits?

THE NATIONAL MATHCLUB

Club Leader Solution Guide!

Defy Superstition Day

1. A horseshoe is thought to hold good luck when it is mounted with the opening facing straight up, as shown. This is how Marge hung the horseshoe on her front door using one nail. However, Kenneth then rotated the horseshoe 135 degrees clockwise on the nail. After that, Abby came along and rotated the horseshoe 240 degrees counterclockwise on the nail. In order to get the horseshoe back to its original position, what is the fewest number of degrees *counterclockwise* Marge would have to rotate the horseshoe?

   Rotating the horseshoe 135 degrees clockwise and 240 degrees counterclockwise is a total rotation of 240 − 135 = 105 degrees counterclockwise. The horseshoe's original position could be considered a 360 degree rotation. So, 360 − 105 = 255 degrees counterclockwise.

2. Cloie was looking for four-leaf clovers. She was able to find three of them in a very small square patch of clover measuring 18 inches on each side. Based on this patch, how many four-leaf clovers would she expect to find in her backyard of clover that measures 30 feet by 45 feet?

   Knowing there are 12 inches in 1 foot, 30 feet is 12 × 30 = 360 inches, and 45 feet is 12 × 45 = 540 inches. Divide the backyard into 18-inch squares by dividing 360 ÷ 18 = 20 and 540 ÷ 18 = 30; 20 × 30 = 600 squares that are 18 in. by 18 in. If 3 four-leaf clovers are in each of these squares, 600 × 3 = 1800 four-leaf clovers.

3. Braxton was born on Friday, Feb. 13, 2004. The year 2004 was a leap year. Though his birthday will always be on the 13th, it won't always be on a Friday the 13th or during a leap year. In what year after 2004 will Braxton's birthday first occur on a Friday the 13th?

   There are 366 days in a leap year. To find when Feb. 13 falls in 2005, take 366 days in the year divided by 7 days in a week to get 52 weeks and 2 days left over (52 r 2). This means Feb. 13 in 2005 would be two days later than Friday (i.e. Sunday). The same process can be used to find what day Feb. 13 falls on in 2006, only this time, use 365 for the total number of days in the year (since it is not a leap year). So, 365 ÷ 7 = 52 r 1. This means Feb. 13 falls one day later in 2006 than in 2005. So, one day later than Sunday is Monday. Continue to do this, keeping in mind that 2008 is also a leap year, and find that Braxton's birthday will first occur on a Friday the 13th in 2009.

4. Alexis was hesitant to walk under a ladder, but then she saw a lucky rabbit's foot on the first rung of the ladder, a lucky penny on the fourth rung, and a lucky four-leaf clover on the fifth rung. The rungs are all equally spaced from each other, and the ladder is on level ground. If the rabbit's foot would fall 8 inches straight to the ground and the penny would fall 32 inches straight to the ground, how far would the four-leaf clover fall straight to the ground, in inches?

   There are 3 rungs between the first rung and the fourth rung, and 32 − 8 = 24 inches. Dividng 24 by 3, find that the rungs are spaced 8 inches apart. So, 32 + 8 = 40 inches for the

height of the fifth rung.

5. Angel knew that blowing out all five of the candles on her birthday cake in one breath would bring her good luck and would ensure that her birthday wish came true. Unfortunately, three of Angel's five candles were still lit after she tried to blow them all out with her first breath. How many different sets of three candles could have been left burning after her first attempt?

   There are 10 possible sets. Assume the candles are numbered 1 through 5. The possible sets are 123, 124, 125, 134, 135, 145, 234, 235, 245 and 345. The order of the candles does not matter.

6. Matlyn was so excited about her new Good Luck Ladybug Umbrella that she completely forgot it was bad luck to open the umbrella inside the house. She sprung it open to show her friend Izzy that not only did her umbrella have a lucky ladybug on each of the 8 equally-sized panels, it also had color-symmetry across the vertical, dark black line when looking at the top view shown. If we know the color of the two panels labeled and each panel of the umbrella is either yellow or blue, how many different ways could the remaining panels be colored?

   The panel opposite the yellow panel, across the vertical black line, must also be yellow, and the panel opposite the blue panel must also be blue. There are 4 ways that the remaining panels can be colored to have color-symmetry. (1) all 4 remaining panels are all blue; (2) all 4 remaining panels are all yellow; (3) the upper two panels are yellow and the bottom two panels are blue; and (4) the upper two panels are blue and the bottom two panels are yellow.

7. Carl saw a penny on the sidewalk that was "heads up," so he knew it would be good luck to pick it up. Then he saw that the year on it was a multiple of 13, so he put the unlucky penny back down. After a couple of seconds, though, he realized the year was also a multiple of lucky 7, so it became lucky again and he picked it back up. If the year on the penny was in the 20th century, what year was on the penny?

   The first year in the 20th century (the 1900s) that is a multiple of both 7 and 13 is 1911.

8. Martin was trying to avoid the black cat he saw walking up ahead, but was distracted by also trying to avoid stepping on any cracks in the sidewalk. As a result, Martin and the black cat actually walked right into each other! The cat had been 15 feet from the point of the collision when Martin was 20 feet from it. Before the collision the cat had been walking due east at a constant rate of 9 mph. If Martin had been walking due north at a constant rate, how fast was Martin walking, in mph, to cause the collision?

   Because Martin and the black cat end up at the same point at the same time, the scenario can be represented by a proportion. $15 / 9 = 20 / r$, where r = Martin's rate in mph. The constant of proportionality between their distances is $20 / 15 = 1\ 1/3$. Applying this constant of proportionality to 9 mph gives $r = 9 \times (1\ 1/3) = 12$ mph.

9. Kendra had a square mirror measuring 9 inches on each side. Unfortunately, she dropped the mirror, and it broke into three pieces, as shown, each of equal area. (Don't worry, if the pieces are all buried in the ground under moonlight, she can avoid her 7 years of bad luck.) What is the length of

the perimeter, in inches, of the four-sided piece? Express your answer as a decimal to the nearest tenth.

The area of the entire mirror is 9 × 9 = 81 inches. If the 3 pieces are each of equal area, then the area of each piece is 81 ÷ 3 = 27 square inches. The area of a triangle is represented by the equation A = ½*bh*, where *b* = the length of the base and *h* = the length of the height. Filling in what is known about each triangle piece of the broken mirror, 27 = ½ × 9 × *b*, which simplifies to 27 = 4.5*b*. Using the inverse operation, *b* = 27 ÷ 4.5 = 6 inches, the length of the short side of each triangle. This means that each of the two short sides of the 4-sided piece is 9 − 6 = 3 inches. To find the length of each of the long sides of the 4-sided piece, use the Pythagorean Theorem on the triangle pieces. $9^2 + 6^2 = L^2$, where *L* = the length of the hypotenuse (also the long side of the 4-sided piece). $L^2 = 81 + 36 = 117$. Using the inverse operation, *L* = the square root of 117 = 10.8. Finally, to find the perimeter of the 4-sided piece, add 10.8 + 10.8 + 3 + 3 = 27.6 inches.

10. The Cougars had only three hits in 24 at-bats during the first eight innings of the game. For the ninth inning, Aaron and his dad put on their rally caps in hopes that the Cougars would come back from a two-run deficit. (Rally caps are baseball caps worn in a silly way, perhaps inside out, to bring good luck to a team.) If the Cougars' first six at-bats in the ninth inning each resulted in hits, and this then ended the game, what percent of the Cougars' at-bats during the entire game resulted in hits?

With the first six at-bats in the ninth inning resulting in hits, the Cougars are up to 3 + 6 = 9 total hits out of 24 + 6 = 30 total at-bats. To find the percent of at-bats that resulted in hits, divide 9 ÷ 30 = 0.3, then multiply by 100 to get = 30%.