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*Activity Sheet for the December, 2019, MATHCOUNTS Mini*

## Warm-Up!

Try these problems before watching the lesson.

1. Samantha walks 5 miles north, 2 miles east, and then one mile north to point $P$. How many miles from her starting point is point $P$? Express your answer in simplest radical form.
2. If two distinct circles and an ellipse are drawn, what is the maximum possible number of points at which at least two of the three curves intersect?
3. Two 8.5-inch by 11-inch sheets of paper are lying flat on an otherwise unoccupied 2-foot by 3-foot tabletop. Exactly 700 $\text{in}^2$ of the table are not covered by the sheets of paper. What is the area of the overlap of the two sheets, in square inches?
4. Circle $O$ has its center at $(-4, 1)$ and a radius of 5 units. What is the sum of the $y$-coordinates of the two points where circle $O$ intersects the $y$-axis?

## The Problems

**First Problem:** A rectangle is twice as long as it is wide. When the lengths of all sides are increased by 3 feet, the area of the new rectangle is triple that of the original rectangle. What is the length of the new rectangle?

**Second Problem:** A canary flies directly east 4000 meters at a speed of 20 m/s. It then immediately turns and flies directly north for 3000 meters at a speed of 30 m/s. The canary then flies back to its starting point in a straight line in 100 seconds. What is the average speed of the canary over the entire trip?

## Follow-up Problems

5. A rectangle has area 108 square inches and perimeter 42 inches. If the length and the width are both increased by 1 inch, then what is the area of the resulting rectangle?

6. Three-fifths of the way up a hill, Jack and Jill realized that they had forgotten their bucket. Jill continued up the hill, while Jack went back down the hill to get the bucket. Two minutes after turning back, Jack reached the bottom of the hill at the exact same time that Jill reached the top. If the total distance from the bottom to the top of the hill is 1260 feet, what is the absolute difference in Jack's downhill speed and Jill's uphill speed, in feet per second? Express your answer as a decimal to the nearest tenth.

7. Triangle $PQR$ is a right triangle with $\angle Q = 90°$, $PQ = 3$ and $QR = 4$. Points $S$, $T$ and $U$ are on sides $\overline{PQ}$, $\overline{PR}$ and $\overline{QR}$, respectively, such that $QSTU$ is a square. Find the length of $\overline{ST}$. Express your answer as a common fraction.

8. A 36-inch rope is cut into three pieces. One piece is five inches longer than another, and one piece is twice as long as another. What is the sum of the possible lengths of the longest piece? Express your answer as a decimal to the nearest tenth.

## Wow! Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).