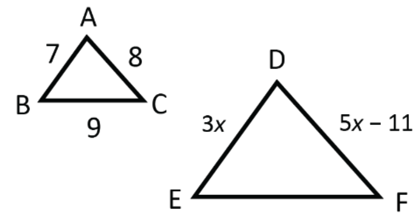
Sprint 1 What is the value of 11 – 18 + 25 – 32 + 39 – 46?

Sprint 2 If there are six apples in a bin, four bins in a bundle, and two bundles in a crate, how many apples are in two crates?

Sprint 3 If triangles ABC and DEF are similar, what is the value of *x*?



Sprint 4 Rocky is collecting acorns. The table shows the total number of acorns Rocky has collected by the end of each week for four weeks. If the pattern continues, what is the total number of acorns Rocky will have collected at the end of week 6?

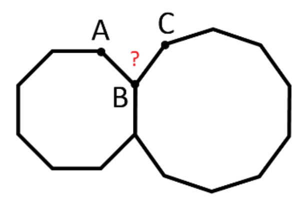
week	1	2	3	4
acorns	6	12	24	48

Sprint 5 Maria is tiling a 10-foot by 4-foot section of her kitchen wall. How many 4-inch by 4-inch square tiles are needed to fully cover the section of wall with no overlapping tiles?

Sprint 6 Vance writes all the four-digit integers that contain each of the digits 1, 4, 8 and 9 exactly once. How many of those integers are even?

Sprint 7 Jakob and Kaleb are typing as many words as they can in a typing race. Jakob is a faster typist, so he gives Kaleb a 24-word head start. If Jakob types 20 words per minute, and Kaleb types 16 words per minute, how many words have they each typed when they have typed the same number of words?

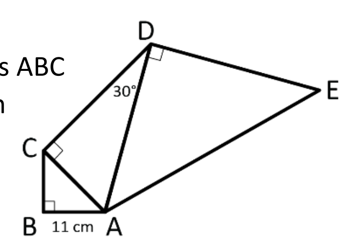
Sprint 8 A regular octagon and a regular decagon share a side, as shown. What is the measure of angle ABC?



Sprint 9 Navi's famous curry powder recipe calls for 3 tsp of ground coriander seeds, 1½ tsp of cumin, 1 tsp of turmeric, and ½ tsp each of ground black pepper, chili powder and ground ginger. What fraction of Navi's curry powder is cumin? Express your answer as a common fraction.

Sprint 10 On Sunday, Cleo starts with a collection of 13 pencils. On Monday, Cleo adds 2 pencils to their collection. On Tuesday, Cleo adds 5 pencils, and then adds 8 pencils on Wednesday. If Cleo continues in this manner, so that the number of pencils added each day is increasing linearly, how many pencils will be in Cleo's collection after pencils are added on the next Sunday?

Sprint 11 Pentagon ABCDE is composed of right triangle ACD and isosceles right triangles ABC and ADE, as shown. If AB = 11 cm and $m\angle ADC$ = 30 degrees, what is the length of segment EA?

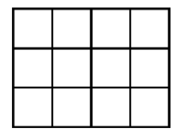


Sprint 12 If the ratio of $3x + 4y$ to $2x + y$ is 18 to 7, what is the ratio of y to x? Express your answer as a common fraction.

Sprint 13 In convex quadrilateral WXYZ, the degree measure of angle X is four times that of angle W. The measure of angle Y is double that of angle X. Lastly, the measure of angle Z is 3 degrees larger than that of angle Y. What is the degree measure of the largest angle in WXYZ?

Sprint 14 Two numbers are randomly chosen from the set {0, 2, 3, 4, 5, 9} without replacement. What is the probability that their product is positive and even? Express your answer as a common fraction.

Sprint 15 How many squares of any size are in the 3 by 4 array of squares shown?



Sprint 16 What will be the time 2022 minutes after 1:18 p.m.?

Sprint 17 Chris is making paper rhombuses, each with perimeter 20 inches and diagonal length 8 inches. If Chris makes 30 such rhombuses, how many square inches of paper are required?

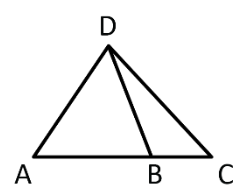
Sprint 18 A sphere and a cylinder have the same radius length and the same volume. What is the ratio of the radius to the height of the cylinder? Express your answer as a common fraction.

Sprint 19 It takes Brandy 6 hours to repair a computer. Lisa can perform the same repairs in 4 hours. Brandy and Lisa spend 1 hour working together to repair a computer. Brandy finishes repairing the computer on her own. What is the total number of hours Brandy spent working to repair the computer? Express your answer as a decimal to the nearest tenth.

Sprint 20 What is the units digit of $20^{20} - 19^{19}$?

Sprint 21 How many two-digit numbers in base 10 are three-digit numbers in base 6?

Sprint 22 Triangle DAC has an area of 80 cm^2. Triangles DAB and DBC both have their bases along the straight segment AC. If AB:BC is 4:1, what is the area of DBC?



Sprint 23 When an analog clock reads 7:50 a.m., what is the measure in degrees of the lesser angle formed by the hour hand and the minute hand?

Sprint 24 Jonathan writes down three positive integers that are perfect cubes. Terence points out that each number is a multiple of 20. What is the smallest possible integer that Jonathan could have written?

Sprint 25 Mira's teacher has written five positive integers on the board, two of which are smudged. The numbers Mira can read are 30, 6 and 24. If all five numbers are less than 100 and their median is 24, what is the greatest possible value of the arithmetic mean of the five numbers? Express your answer as a decimal to the nearest tenth.

Sprint 26 Let $x \$ y = x^2 + y^2 + x + y$, and $3 \$ y = 42$. What is the value of $7 \$ y$?

Sprint 27 What is the area of quadrilateral ABCD with vertices A(4, 6), B(4, 3), C(−2, 1) and D(−5, 6)? Express your answer as a decimal to the nearest tenth.

Sprint 28 What is the greatest five-digit number whose digits have a product of 2160?

Sprint 29 A bag contains maroon marbles and gold marbles. The probability of pulling two maroon marbles without replacement is 7/15. The probability of pulling two gold marbles without replacement is 1/15. What is the least number of marbles that could possibly be in the bag?

Sprint 30 Jack has found a treasure chest filled with over 200 gold coins. If Jack separates the coins into 4 equal piles, there is 1 coin left over. If Jack separates the coins into 7 equal piles, there is 1 coin left over. If Jack separates the coins into 9 equal piles, there are 2 coins left over. What is the least possible number of coins in the treasure chest Jack found?

Target 1 Sydra has a bedside table with a lamp. On the same table, a figurine that is 6 inches tall casts a shadow that is 8 inches long. Sydra replaces the figurine with a water bottle that is 9 inches tall. How long is the shadow that the water bottle casts?

Target 2 Caden has three indistinguishable \$1 bills. In how many ways can Caden distribute all three \$1 bills among three of his friends?

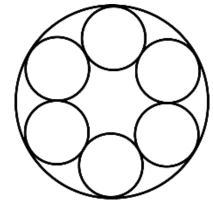
Target 3 A triangle has two sides of lengths 8 and 13. How many possible integer lengths are there for the third side?

Target 4 What is the tens digit of the difference $2021^{2021} – 2020^{2020}$?

Target 5 Vineesh and Sarayu raced around a track from the same starting point. Sarayu ran, on average, 30% faster than Vineesh. If Sarayu finished the race 24 seconds ahead of Vineesh, how many seconds did it take Sarayu to finish the race?

Target 6 Four fair six-sided dice are rolled. What is the probability that at least two dice show a 3? Express your answer as a common fraction.

Target 7 A large circle contains six small circles, each of which is tangent to two other small circles and the large circle, as shown. If each small circle has a radius of 4 cm, what is the area of the large circle? Express your answer as a decimal to the nearest tenth.



Target 8 Chelsea notices that the sum of the positive factors of a positive integer n is equal to $2.8n$. What is the least possible value of n?

Team 1 How many of the numbers from 1 to 100, inclusive, are multiples of 3 or 5?

Team 2 The numbers of seats in consecutive rows of an auditorium form an arithmetic sequence. The first row has 17 seats; the second row has 23 seats; and the third row has 29 seats. How many seats are in the 10th row?

Team 3 Myles is buying food from a concession stand that only sells hotdogs and hamburgers. Hotdogs cost \$1.50 each, and hamburgers cost \$3.50 each. Myles must buy at least 3 hamburgers. If he has \$20 to spend, what is the maximum number of items that he can buy from this concession stand?

Team 4 There are 280 students at a school. If 2/7 of the students play basketball and 15% of the students play soccer, and 17 students play both, how many students play neither?

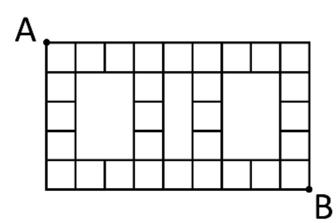
Team 5 A right circular cone has a base radius of 6 meters and a slant height of 12 meters. This cone is cut parallel to its base to form a smaller cone and a frustum. The volume of the frustum is 1/3 that of the original cone. What is the volume of the frustum? Express your answer to the nearest whole number of cubic meters.

Team 6 For the two-digit numbers A3 and 4B, where A and B represent digits, A3 × 4B = 2021. What is the value of A + B?

Team 7 Nestor writes down five consecutive positive integers. He then notices that two of his five integers add up to k, while the other three integers also add up to k. What is the greatest possible value of k?

Team 8 Three distinct numbers are randomly selected from the set {1, 2, 3, 6, 7, 9, 12, 14} and multiplied together. What is the probability that the product of these three numbers has an odd number of factors? Express your answer as a common fraction.

Team 9 In the figure shown, how many paths are there from A to B moving only down and to the right?



Team 10 There is exactly one ordered pair of positive integers (x, y) that satisfies the equation $57x + 29y = 1234$. What is the value of $x + y$?

2022-2023 Practice Competition — Answers

Sprint 1	−21
Sprint 2	96 apples
Sprint 3	7
Sprint 4	192 acorns
Sprint 5	360 tiles
Sprint 6	12 integers
Sprint 7	120 words
Sprint 8	81 degrees
Sprint 9	3/14
Sprint 10	90 pencils
Sprint 11	44 cm
Sprint 12	3/2
Sprint 13	139 degrees
Sprint 14	7/15
Sprint 15	20 squares
Sprint 16	11:00 p.m. or 11 p.m.
Sprint 17	720 in^2
Sprint 18	3/4
Sprint 19	4.5 hours
Sprint 20	1
Sprint 21	64 numbers
Sprint 22	16 cm^2
Sprint 23	65 degrees
Sprint 24	1000
Sprint 25	36.6
Sprint 26	86
Sprint 27	31.5 $units^2$
Sprint 28	98,651
Sprint 29	10 marbles
Sprint 30	281 coins

Target 1	12 inches
Target 2	10 ways
Target 3	15 lengths
Target 4	2
Target 5	80 seconds
Target 6	19/144
Target 7	452.4 cm^2
Target 8	60

Team 1	47 numbers
Team 2	71 seats
Team 3	9 items
Team 4	175 students
Team 5	131 m^3
Team 6	11
Team 7	15
Team 8	5/56
Team 9	304 paths
Team 10	30

Sprint 1

We can add parentheses to see that each pair of terms has a difference of –7: (11 – 18) + (25 – 32) + (39 – 46) = –7 + (–7) + (–7) = **–21**.

Sprint 2

We can multiply (6 apples/bin) × (4 bins/bundle) × (2 bundles/crate) to find that there are 48 apples/crate. So, in two crates, there are 48 × 2 = **96** apples.

Sprint 3

Because these triangles are similar, we can set up and solve a proportion to find the value of x: $(3x)/7 = (5x - 11)/8 \rightarrow (3x) \times 8 = (5x - 11) \times 7 \rightarrow 24x = 35x - 77 \rightarrow -11x = -77 \rightarrow x =$ **7**.

Sprint 4

We can see that each week, the number of acorns Rocky has collected doubles. So, at the end of week 5, Rocky will have collected 48 × 2 = 96 acorns, and at the end of week 6, Rocky will have collected 96 × 2 = **192** acorns. Alternatively, we are looking for the final number of acorns after 6 weeks, which is 2 more weeks from what we are given in the table. So, we can multiply the number of acorns at the end of week 4 by $2 \times 2 = 2^2$ to get $48 \times 2^2 =$ **192** acorns.

Sprint 5

Maria is tiling a section of her kitchen wall that is 10 × 4 = 40 ft^2. One square foot is 12 inches by 12 inches. Since 12 inches ÷ 4 inches = 3, Maria would be able to fit 3 × 3 = 9 tiles in a 1-ft^2 section. So, 9 tiles/ft^2 × 40 ft^2 = **360** tiles would be needed to cover this section of Maria's kitchen wall.

Sprint 6

For the number to be even, the units digit must be either 4 or 8, so there are 2 options for the units digit. The placement of the remaining digits does not affect whether the number is even, so for each of the two options of units digit, there are 3! = 3 × 2 × 1 = 6 ways to place the remaining three digits. Thus, of all the four-digit integers containing these digits, 2 × 6 = **12** integers are even.

Sprint 7

We can represent the number of words typed by Jakob with the equation $J = 20m$, where m is the number of minutes. We can represent the number of words typed by Kaleb with the equation $K = 16m + 24$. We're looking for how many words they each typed when they have typed the same number of words, in other words, when $J = K$. We can set these expressions equal to each other and solve for m: $20m = 16m + 24 \rightarrow 4m = 24 \rightarrow m = 6$. This means that after 6 minutes, Jakob and Kaleb have typed the same number of words. So, they have each typed 20(6) = 16(6) + 24 = **120** words.

Sprint 8

An interior angle of a regular convex figure can be calculated using the formula $[180(n - 2)]/n$, where n is the number of sides of the figure. In a regular octagon, there are [180(8 – 2)]/8 = 180(6)/8 = 135 degrees in each interior angle. In a regular decagon, there are [180(10 – 2)]/10 = 180(8)/10 = 144 degrees in each interior angle. We know that a point has 360 degrees around it. In the 360 degrees around point B, there is an angle from a regular octagon, an angle from a regular decagon, and angle ABC (whose measure we'll call x): $135 + 144 + x = 360 \rightarrow 279 + x = 360 \rightarrow x = 81$. Thus, the measure of angle ABC is **81** degrees.

Alternatively, if we extend the line containing the shared side, angle ABC is divided into two angles. One of the angles is the supplement of the interior angle of the octagon and has measure 180 – 135 = 45 degrees. The other angle is the supplement of the interior angle of the decagon and has measure 180 – 144 = 36 degrees. Therefore, the measure of angle ABC is 45 + 36 = **81** degrees.

Sprint 9

Adding up the amounts of all the spices, we get 3 + 1.5 + 1 + 3(0.5) = 7 total teaspoons of spices. The curry powder is 1.5/7 = **3/14** of the mix.

Sprint 10

Cleo starts off with 13 pencils, adds 2, followed by 5, then 8. The number of pencils Cleo is adding to the collection each day is increasing by 3. So, continuing the pattern through the next Sunday, Cleo will have 13 + 2 + 5 + 8 + 11 + 14 + 17 + 20 = **90** pencils.

Sprint 11

Triangle ABC is an isosceles right triangle, which means it is a 45-45-90 right triangle. We know the legs of triangle ABC each have length 11 cm, so by properties of 45-45-90 right triangles, the hypotenuse of triangle ABC has length $11 \times \sqrt{2} = 11\sqrt{2}$ cm. Triangle ACD is a 30-60-90 right triangle with a shorter leg of length $11\sqrt{2}$ cm. By properties of 30-60-90 right triangles, the hypotenuse AD has length $2 \times 11\sqrt{2} = 22\sqrt{2}$ cm. Finally, triangle ADE is also an isosceles right triangle, which makes it a 45-45-90 right triangle with legs of length $22\sqrt{2}$ cm. By properties of 45-45-90 right triangles, hypotenuse AE has length $22\sqrt{2} \times \sqrt{2} =$ **44** cm. Alternatively, we can use the Pythagorean theorem to find the length of AC: $11^2 + 11^2 = AC^2 \rightarrow 2(11^2) = AC^2 \rightarrow AC = 11\sqrt{2}$ cm. Using 30-60-90 triangle ratios, we know that the length of AD must be double the length of AC, so $AD = 2(11\sqrt{2}) = 22\sqrt{2}$ cm. Applying the Pythagorean theorem again to triangle ADE to find the length of EA, we find: $(22\sqrt{2})^2 + (22\sqrt{2})^2 = EA^2 \rightarrow 2(22\sqrt{2})^2 = EA^2 \rightarrow EA = 2 \times 22 =$ **44** cm.

Sprint 12

Given the proportion $(3x + 4y)/(2x + y) = 18/7$, we can cross-multiply to find that $18(2x + y) = 7(3x + 4y) \rightarrow 36x + 18y = 21x + 28y \rightarrow -10y = -15x \rightarrow y/x = -15/(-10) =$ **3/2**.

Sprint 13

Let w, x, y and z represent the degree measures of angles W, X, Y and Z, respectively. The interior angle sum of a quadrilateral is 360 degrees. Thus, $w + x + y + z = 360$. We can represent the provided information with the following equations: $y = 2x$, $x = 4w$ and $z = y + 3$. Because we do not know anything about w, let's rewrite each equation in terms of w, so that we are able to solve for it. We already have $x = 4w$. Next, $y = 2(4w) = 8w$. Then, $z = 8w + 3$. Substituting each of these equations into $w + x + y + z = 360$, we get $w + 4w + 8w + (8w + 3) = 360 \rightarrow 21w + 3 = 360 \rightarrow 21w = 357 \rightarrow w = 17$ degrees. We are asked for the measure of the largest angle, which is $z = 8w + 3 = 8(17) + 3 =$ **139** degrees.

Sprint 14

Randomly selecting 2 numbers from a set of 6 numbers, without replacement, results in ${}_6C_2 = 6!/[2!(6-2)!] = 6!/[2!4!] = (6 \times 5)/(2 \times 1) = 15$ outcomes. Specifically, we want to know how many of these outcomes are two numbers whose product is positive and even. Because the product must be positive, we know that neither of the chosen numbers can be 0. In order to have an even product, at least one of the numbers chosen must be even. In the given set, there are two even numbers (2 and 4), so there are 2 choices for the first number. Then, there are 4 numbers left in the set from which to choose the second number. This gives 2 × 4 = 8 possible outcomes in which the product of the two numbers chosen is positive and even. However, in this method, we have counted the selection of 4 and 2 twice (2, then 4 and 4, then 2). So, we must subtract 1 to get 8 – 1 = 7 favorable outcomes. Therefore, the probability of this happening is **7/15**.

Sprint 15

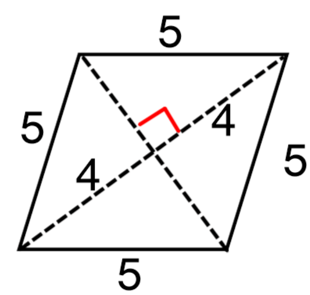
There are 12 squares that are 1 × 1. There are 6 squares that are 2 × 2. Finally, there are 2 squares that are 3 × 3. This is a total of 12 + 6 + 2 = **20** squares.

Sprint 16

We can divide 2022 into groups of 60 to represent the number of hours in 2022 minutes. We find that 60 goes in 33 times with a remainder of 42 minutes. With 24 hours in one full day, we must add 33 – 24 = 9 hours and 42 minutes to 1:18 p.m. This gives us **11:00** p.m.

Sprint 17

The area of a rhombus is $(d_1d_2)/2$, where d_1 and d_2 are the lengths of its diagonals. Given that the perimeter is 20 inches, each side length must be 5 inches. We also know that the diagonals in a rhombus are perpendicular and bisect each other. So, we can divide the 8-inch diagonal into two segments, each of length 4 inches, as shown. Now, we have four congruent right triangles. Recognizing that each of these triangles is a 3-4-5 right triangle, we can see that the other diagonal is divided into two segments, each of length 3 inches. Thus, this diagonal must have length 3 + 3 = 6 inches. Therefore, the area of one of these rhombuses is (8 × 6)/2 = 24 in^2. Chris is making 30 such rhombuses, so he will need 30(24) = **720** in^2 of paper.



Sprint 18

The volume of a sphere of radius r is $(4/3)\pi r^3$. The volume of a cylinder of radius r and height h is πr^2h. Here, the figures have the same volume and radius length, so $(4/3)\pi r^3 = \pi r^2h$. Simplifying, we get $(4/3)r = h \rightarrow r = (3/4)h \rightarrow r/h =$ **3/4**.

Sprint 19

Brandy can perform 1/6 of the computer fixes per hour, and Lisa can perform 1/4 of the computer fixes per hour. We can represent this scenario as $(1/6)(t + 1) + (1/4)(1) = 1$, where t is the number of additional hours Brandy worked on the computer. Solving for t, we get $(1/6)t + 1/6 + 1/4 = 1 \rightarrow 2t + 2 + 3 = 12 \rightarrow 2t + 5 = 12 \rightarrow 2t = 7 \rightarrow t = 7/2 = 3.5$ hours. But, we need to add the additional hour Brandy worked with Lisa, so this gives a total of 3.5 + 1 = **4.5** hours.

Sprint 20

The units digit of 20^{20} is 0, and 20^{20} is larger than 19^{19}. The units digit of 19^n is 9 on odd powers and 1 on even powers, so the units digit of 19^{19} is 9. When subtracting $20^{20} – 19^{19}$, we are subtracting 9 from 0 in the units place, so after borrowing from the tens place, the result will be 10 – 9 = **1**.

Sprint 21

Looking at two-digit numbers in base 10, we have numbers 10 through 99, inclusive. A three-digit number in base 6 means that in base 10, it must be greater than or equal to 36 but less than 216. So, the numbers 36 through 99, inclusive, fit both conditions. Therefore, there are 99 – 36 + 1 = **64** two-digit numbers in base 10 that are three-digit numbers in base 6.

Sprint 22

Because both triangles have the same height, the ratio of the bases also applies to the area. So, since BC/AC = 1/(1 + 4) = 1/5, the ratio of the area of DBC to that of triangle ADC is 1/5 as well. Thus, the area of DBC is (1/5) × 80 = **16** cm^2.

Sprint 23

Every hour, the hour hand moves 360/12 = 30 degrees, which means it moves 30/60 = 0.5 degrees per minute. The minute hand moves 360/60 = 6 degrees per minute. By 7 a.m., the hour hand has rotated 30 × 7 = 210 degrees clockwise from where it started at midnight. By 7:50, the hour hand has moved an additional 50 × 0.5 = 25 degrees for a total of 210 + 25 = 235 degrees clockwise from where it started at midnight. After 50 minutes, the minute hand has moved 6 × 50 = 300 degrees clockwise from where it started at midnight. The difference in the numbers of degrees rotated by the minute and hour hands is the measure of the lesser angle formed by the hands. That angle measure is 300 – 235 = **65** degrees.

Sprint 24

We know that Jonathan's integers are all multiples of 20 and are perfect cubes. As multiples of 20, they must be divisible by $2^2 \times 5$. However, as perfect cubes, each power must at least be a multiple of 3. Therefore, the smallest possible integer Jonathan could have written is $2^3 \times 5^3$ = **1000**.

Sprint 25

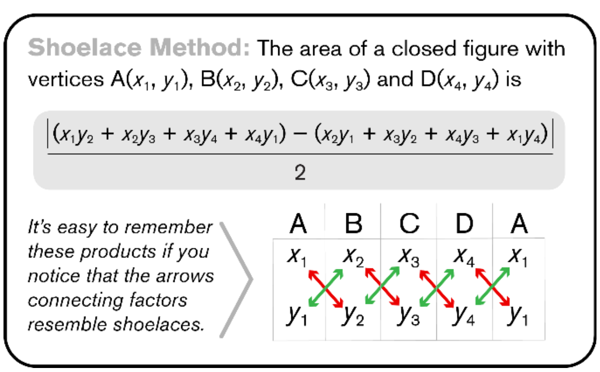
In order to find the greatest possible mean of the five numbers, we want to fill in the missing numbers with the greatest possible values, which must be one- or two-digit numbers. Knowing that 24 is the median of the five numbers, we can say that the order 6, __, 24, 30, __ will satisfy the conditions and lead to the largest possible mean. Let's put 99, the largest possible value, in the right-most blank. Because the median is 24, 24 is the largest possible value for the left-most blank. This gives us a list of 6, 24, 24, 30, 99, which has a mean of **36.6**.

Sprint 26

Because $x \$ y = x^2 + y^2 + x + y$ and $3 \$ y = 42$, we know $3^2 + y^2 + 3 + y = 42$, which can be rewritten as $y^2 + y - 30 = 0$. This factors to $(y + 6)(y - 5) = 0$. Therefore, y equals either –6 or 5. The problem doesn't specify if y is positive or negative, so we should be able to substitute either value for y in the expression $7 \$ y$ to find its value – let's try them both out. First, substituting 5 for y yields $7^2 + 5^2 + 5 + 7 = 86$. Substituting –6 for y yields $7^2 + (-6)^2 + (-6) + 7 = 86$, as well. So, $7 \$ y$ = **86**. Alternatively, we find that $7 \$ y = y^2 + y + 56$. We also know that $y^2 + y = 30$ from our work earlier with $3 \$ y$. Substituting 30 for $y^2 + y$, we get $7 \$ y = y^2 + y + 56 = 30 + 56$ = **86**.

Sprint 27

The easiest way to do this is by using the Shoelace Method, a formula for finding the area of a closed figure using the coordinates of its vertices.



So, using the Shoelace Method, we calculate |[(4 × 3) + (4 × 1) + (–2 × 6) + (–5 × 6)] – [(6 × 4) + (3 × (–2)) + (1 × (–5)) + (6 × 4)]|/2 = |12 + 4 – 12 – 30 – 24 + 6 + 5 – 24|/2 = |15 – 78|/2 = |–63|/2 = 63/2 = 31.5. Therefore, the area of quadrilateral ABCD is **31.5** units^2.

Sprint 28

The prime factorization of 2160 is $2^4 \times 3^3 \times 5$. We are looking for the largest possible five-digit number, so the leading digit should be $9 = 3^2$. Now, we only have one factor of 3 left, so the next highest digit we can have is $8 = 2^3$. So, we have 98,_ _ _. At this point, we are left with the following prime factors of 2160: 2, 3, 5. We can use 6 = 2 × 3 as the next highest digit, and then use the remaining prime factor 5. Finally, as we don't have any prime factors of 2160 left, we use a 1 as the units digit to get **98,651**.

Sprint 29

Let's say there are x maroon marbles and y gold marbles. We can represent the probability of pulling two maroon marbles without replacement with the following equation: $[x(x-1)]/[(x+y)(x+y-1)] = 7/15$. Then, we can represent the probability of pulling two gold marbles without replacement with the following equation: $[y(y-1)]/[(x+y)(x+y-1)] = 1/15$. When we cross multiply the first equation, we get $15x(x-1) = 7(x+y)(x+y-1)$. When we cross multiply the second equation, we get $15y(y-1) = (x+y)(x+y-1)$. We can substitute the expression on the left side of this equation in for $(x+y)(x+y-1)$ in the equation $15x(x-1) = 7(x+y)(x+y-1)$ to get $15x(x-1) = 7[15y(y-1)]$, which simplifies to $x(x-1) = 7y(y-1)$. Now, let's do some casework. We know y can't be 1, because that would result in $1(1-1) = 1(0) = 0$. Instead, let's try 2 for y. This gives us $x(x-1) = 7(2)(1) \rightarrow x(x-1) = 14$. However, this doesn't work either, because no two consecutive integers multiply together to get 14. If y is 3, then we get $x(x-1) = 7(3)(2) \rightarrow x(x-1) = 42$. This works since 6 × 7 = 42. So, $x = 7$. We are looking for the least number of marbles that could possibly be in the bag, which would be $x + y = 7 + 3 =$ **10** marbles.

Sprint 30

With a remainder of 1 when dividing by both 4 and 7, and with the fact that these two numbers are relatively prime, we can equate the least possible number of coins in the treasure chest to 1 mod 28. So, we can say that we're looking for a number with a remainder of 1 when divided by 28 and a remainder of 2 when divided by 9. Knowing that there are over 200 coins, and $200 \equiv 2 \bmod 9$, we can see that 281 will have a remainder of 1 when divided by 28 and a remainder of 2 when divided by 9. Therefore, the least possible number of coins in the treasure chest is **281** coins.

Target 1

We can set up a proportion to represent this situation: (6 inches)/(8 inches) = (9 inches)/(x inches), where x is the length of the water bottle's shadow. Cross multiplying, we find $6x = 8 \times 9 \rightarrow 6x = 72 \rightarrow x =$ **12** inches. Alternatively, we can see that there is a 3/4 ratio between the figure and its shadow. We can apply this ratio to the water bottle to find that the length of the water bottle's shadow is $(3/4)x = 9 \rightarrow x =$ **12** inches.

Target 2

Let's use the Stars and Bars Method to solve this problem. We must divide the three \$1 bills into 3 groups, so we will have 2 dividers (or "bars") to separate the groups. With the three \$1 bills (or "stars"), we have 2 + 3 = 5 spots in which to distribute our stars and bars. We can calculate either ${}_2C_5$ or ${}_3C_5$, to find the number of ways we can distribute either the bars or the stars, which both give us $5!/(2! \times 3!) = (5 \times 4)/(2 \times 1) = 20/2 =$ **10** ways to distribute the \$1 bills.

Target 3

If the triangle must have integer side lengths, then we can use the Triangle Inequality Theorem, which states that if the side lengths of a triangle are a, b and c, where $a \le b \le c$, then $a + b > c$. If we set our missing side as c, then c must be less than 8 + 13 = 21. If we set our missing side as a, then $a + 8 > 13$, so a must be greater than 13 – 8 = 5. This results in possible integer side lengths strictly between 5 and 21, which gives us **15** possible lengths.

Target 4

With 2020^{2020}, we know the tens digit and units digit both will be zero. So, we only need to consider 2021^{2021}. In looking for a pattern, we find $21^1 \bmod 100 \equiv 21$; $21^2 \bmod 100 \equiv 41$; $21^3 \bmod 100 \equiv 61$; $21^4 \bmod 100 \equiv 81$; $21^5 \bmod 100 \equiv 01$; and $21^6 \bmod 100 \equiv 21$. We see that the pattern repeats every 5 powers, so we can take 2021 mod 5, which is equivalent to 1. Therefore, the tens and units digits of $2021^{2021} – 2020^{2020}$ are $21^1 – 00 = 21$, so the tens digit of this difference is **2**.

Target 5

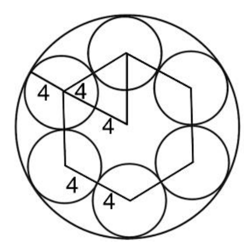
We'll need to use the equation distance (d) = rate (r) × time (t) here. Let Sarayu's time be t. If Vineesh's rate is r, then Sarayu's rate is $1.3r$, since she runs 30% faster than Vineesh. Thus, we can represent Sarayu's race as $d = 1.3rt$. We are told that Sarayu finished the race 24 seconds ahead of Vineesh, so let Vineesh's time be $t + 24$. Thus, Vineesh's race can be represented by $d = r(t + 24)$. We can set these two expressions for d equal to each other to get $1.3rt = r(t + 24) \rightarrow 1.3rt = rt + 24r \rightarrow 0.3rt = 24r \rightarrow 0.3t = 24 \rightarrow t = 80$. Since t represents Sarayu's time, this tells us that Sarayu finished the race in **80** seconds.

Target 6

We know the probability of rolling a 3 is 1/6. If we try to find the probability of getting at least two 3s, we would need to find the probability of getting two 3s, three 3s and four 3s. It is easier to find the complement, which is zero 3s or one 3, and subtract from 1 (100%). To roll zero 3s, each of the four dice could land on any other number, so the probability of this happening is (5/6) × (5/6) × (5/6) × (5/6) = 625/1296. The probability of rolling exactly one 3 is (1/6) × (5/6) × (5/6) × (5/6) × 4 = 125/324. (We need to multiply by 4 here, because the one 3 could appear on any of the four dice, so we must account for each of these scenarios.) Therefore, the probability of rolling at least two 3s is 1 – (625/1296 + 125/324) = 1 – (1125/1296) = 171/1296 = **19/144**.

Target 7

We can form a regular hexagon using the centers of the small circles as its vertices, as shown. Each side of this hexagon is 8 cm in length, since the radius of each of the small circles is 4 cm, and each side of the hexagon is two of these radii together. Because this is a regular hexagon, we also know that connecting the diagonals of the hexagon will form 6 equilateral triangles that have the same side length as the hexagon (one such triangle is shown). Therefore, the radius of the large circle is 4 + 8 = 12 cm. From there, we can calculate the area of the large circle to be $\pi r^2 = \pi(12)^2 = 144\pi \approx$ **452.4** cm^2.



Target 8

Since the sum of the factors is $2.8n$, n must be a multiple of 5. If $n = 5k$, then the sum of the factors must be $5k \times 2.8 = 14k$, which is a multiple of 7. It is not so easy to get the sum of factors divisible by 7. Testing small possibilities reveals that 5 has factor sum 1 + 5 = 6; 10 has factor sum 1 + 2 + 5 + 10 = 18; 15 has factor sum 1 + 3 + 5 + 15 = 24; and 20 has factor sum 1 + 2 + 4 + 5 + 10 + 20 = 42, which is divisible by 7, but 2.8 × 20 does *not* equal 42. Testing more multiples of 5 reveals that most of them don't have a factor sum divisible by 7: 25 has factor sum 1 + 5 + 25 = 31; 30 has factor sum 1 + 2 + 3 + 5 + 6 + 10 + 15 + 30 = 72; 35 has factor sum 1 + 5 + 7 + 35 = 48; 40 has factor sum 1 + 2 + 4 + 5 + 8 + 10 + 20 + 40 =

90; 45 has factor sum 1 + 3 + 5 + 9 + 15 + 45 = 78; 50 has factor sum 1 + 2 + 5 + 10 + 25 + 50 = 93; and 55 has factor sum 1 + 5 + 11 + 55 = 72. None of these are multiples of 7. But, **60** has factor sum 1 + 2 + 3 + 4 + 5 + 6 + 10 + 12 + 15 + 20 + 30 + 60 = 28 × 6 = 2.8 × 60 = 168.

Team 1

Since 100/3 = 33.33333…, we know that there are 33 numbers from 1 to 100, inclusive, that are divisible by 3. Since 100/5 = 20, there are 20 numbers from 1 to 100, inclusive, that are divisible by 5. However, we have double-counted the numbers that are divisible by both. We need to see how many of those numbers are divisible by 3 × 5 = 15. Since 100/15 = 6.66666…, we know that there are 6 numbers divisible by both 3 and 5 from 1 to 100, inclusive. So, there are 33 + 20 – 6 = **47** numbers from 1 to 100, inclusive, that are multiples of 3 or 5.

Team 2

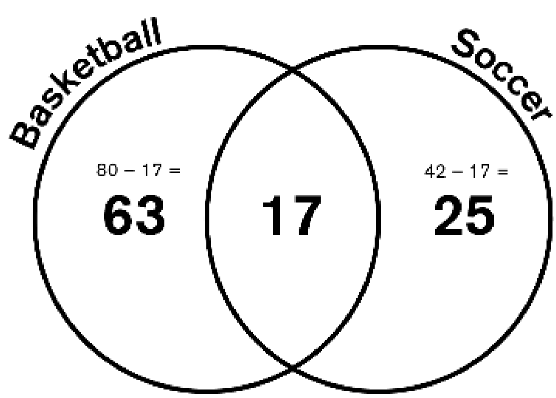
Because the numbers of seats in consecutive rows is an arithmetic sequence, the number of seats increases by the same amount each time. In looking at the pattern, we see that the number of seats increases by 6 in each consecutive row. If we start with the 17 seats in the first row, we can add 6 seats a total of 9 times to get the number of seats in the 10^{th} row. So, there are 17 + 6(9) = **71** seats in the 10^{th} row.

Team 3

If Myles wants to maximize the number of items bought, he should minimize the number of hamburgers, since they are the most expensive item. Since Myles must buy at least 3 hamburgers, let's say he buys exactly 3 hamburgers and therefore spends 3(3.50) = $10.50. Then, he has 20 – 10.50 = $9.50 left over to spend on hotdogs. Dividing $9.50 by the price of a hotdog, $1.50, gives 6 1/3 hotdogs. However, Myles cannot buy part of a hotdog, so he can buy 6 hotdogs. Therefore, the maximum number of items Myles can buy from this concession stand is 3 + 6 = **9** items.

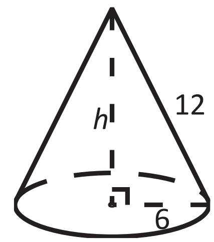
Team 4

With 280 students at the school, 280(2/7) = 80 of them play basketball and 280(0.15) = 42 of them play soccer. Knowing also that 17 students play both, we can create the Venn Diagram shown:



Now, we can subtract the students who play basketball, soccer and both from the total 280 students at the school to find the number of students who don't play either sport: 280 – 63 – 17 – 25 = **175** students. Alternatively, we could simply subtract 280 – 80 – 42 = 158. However, here, we have subtracted the 17 students who play both sports twice. So, we would need to add 17 students back in to find that 158 + 17 = **175** students play neither sport.

Team 5



As the figure shows, the base radius and height of the cone form the legs of a right triangle, and the slant height is the hypotenuse. We can use the Pythagorean theorem to determine the cone's height, but since the base radius is half the length of the slant height, we know this right triangle is a 30-60-90 right triangle. By properties of 30-60-90 right triangles, $h = 6\sqrt{3}$ meters. Now we can use the formula for the volume of a cone to calculate its volume. We have $(1/3)\pi r^2 h = (1/3)\pi(6^2)(6\sqrt{3}) = 72\pi\sqrt{3}$ m^3. We are told the volume of the frustum is 1/3 that of the original cone, so the volume of the frustum is $(1/3) \times 72\pi\sqrt{3} \approx$ **131** m^3.

Team 6

To multiply A3 and 4B, we start with the units digits, where multiplying 3 by B results in a units digit of 1. The only digit by which 3 can be multiplied to give a units digit of 1 is 7 ($3 \times 7 = 21$). So, B = 7. Now, we can take $2021 \div 47 = 43$, which means A = 4. Therefore, A + B = 4 + 7 = **11**.

Team 7

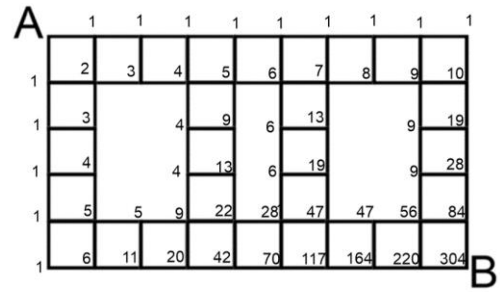
We can represent Nestor's five consecutive numbers as x, $x + 1$, $x + 2$, $x + 3$ and $x + 4$. To maximize the sum k, let's say the group of two integers that add to k are the largest numbers, $x + 3$ and $x + 4$, and the group of three integers that add to k are x, $x + 1$ and $x + 2$. Then, $x + 3 + x + 4 = x + x + 1 + x + 2 \rightarrow 2x + 7 = 3x + 3 \rightarrow x = 4$. With this, we find that the value of k is $2x + 7 = 3x + 3 = 2 \times 4 + 7 = 3 \times 4 + 3 =$ **15**. We can verify that this is the largest possible value of k by looking at the other possible groupings of the five consecutive numbers. Ultimately, the higher the value of x, the higher the sum k.

Group of 2 #s	Group of 3 #s	Equation	Value of x
$x, x + 1$	$x + 2, x + 3, x + 4$	$2x + 1 = 3x + 9$	$x = -8$
$x, x + 2$	$x + 1, x + 3, x + 4$	$2x + 2 = 3x + 8$	$x = -6$
$x, x + 3$	$x + 1, x + 2, x + 4$	$2x + 3 = 3x + 7$	$x = -4$
$x, x + 4$	$x + 1, x + 2, x + 3$	$2x + 4 = 3x + 6$	$x = -2$
$x + 1, x + 2$	$x, x + 3, x + 4$	$2x + 3 = 3x + 7$	$x = -4$
$x + 1, x + 3$	$x, x + 2, x + 4$	$2x + 4 = 3x + 6$	$x = -2$
$x + 1, x + 4$	$x, x + 2, x + 3$	$2x + 5 = 3x + 5$	$x = 0$
$x + 2, x + 3$	$x, x + 1, x + 4$	$2x + 5 = 3x + 5$	$x = 0$
$x + 2, x + 4$	$x, x + 1, x + 3$	$2x + 6 = 3x + 4$	$x = 2$
$x + 3, x + 4$	$x, x + 1, x + 2$	$2x + 7 = 3x + 3$	$x = 4$

Team 8

For the product of these three numbers to have an odd number of factors, it must be a perfect square. We have 8 numbers in the set and are choosing 3 numbers to multiply together. The potential triples with perfect square products are (1, 3, 12), (2, 3, 6), (2, 7, 14), (2, 6, 12) and (3, 9, 12). Therefore, there are 5 possible outcomes of randomly selecting three distinct numbers from the set that would result in a product with an odd number of factors. There are ${}_8C_3 = 8!/(3!(8 - 3)!) = 8!/(3!5!) = (8 \times 7 \times 6)/(3 \times 2 \times 1) = 336/6 = 56$ ways to select three distinct numbers from the given set. So, the requested probability is **5/56**.

Team 9



We can only move down or to the right, which means that there will be exactly 1 path along all edges on the top row and left column. From there, we move diagonally, keeping track of the number of paths to each intersection by adding the numbers of paths to the previous intersections to get the number of paths to each new intersection, as shown, to get **304** paths.

Team 10

Because 29 is the smaller coefficient of the *x* and *y* terms, let's take the whole equation mod 29. Doing so gives us $28x \equiv 16 \bmod 29$. We know that $28 \equiv -1 \bmod 29$, so $-x \equiv 16 \bmod 29$ or $x \equiv -16 \bmod 29$. The problem states both *x* and *y* must be positive integers, so we add 29 to –16 to get $x = 13$. Substituting this value back into our original equation gives $57(13) + 29y = 1234 \rightarrow 741 + 29y \rightarrow 29y = 493 \rightarrow y = 17$. So, $x + y = 13 + 17 =$ **30**.