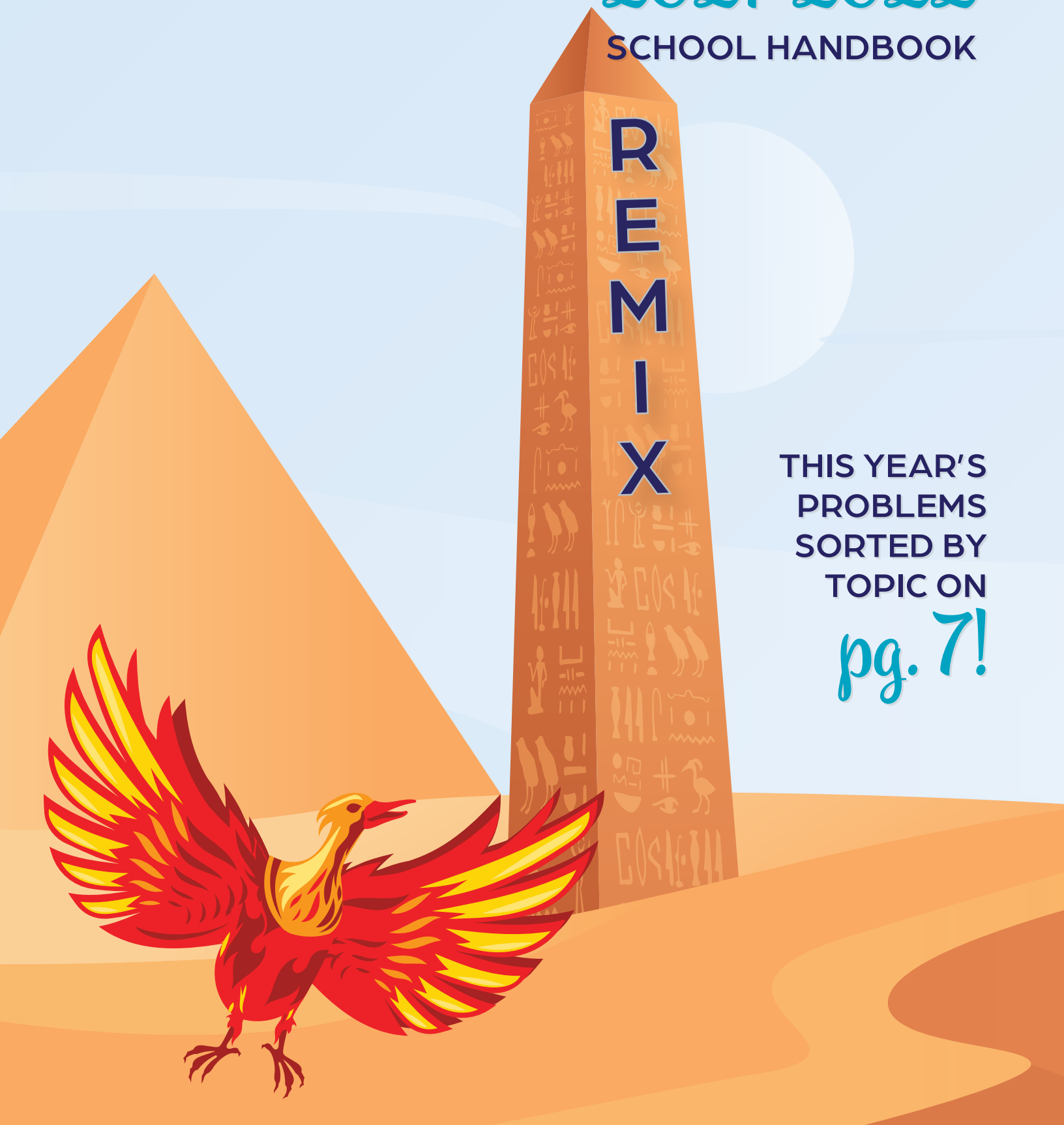
MATHCOUNTS®

2021-2022

SCHOOL HANDBOOK



THIS YEAR'S PROBLEMS SORTED BY TOPIC ON pg. 7!

2021-2022 MATHCOUNTS® School Handbook Remix

Title Sponsors:
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National Council of Teachers of Mathematics
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WestEd has recognized MATHCOUNTS as having one of the nation's most effective STEM learning programs, listing the Math Video Challenge as an Accomplished Program in STEMworks.

The National Association of Secondary School Principals has placed all three MATHCOUNTS programs on the NASSP Advisory List of National Contests and Activities for 2021-2022.

TABLE OF CONTENTS

We're excited you're participating in a MATHCOUNTS program this year!
We hope this experience will be a meaningful and enriching one for you and your Mathletes! If you have any questions during the year, please contact the MATHCOUNTS national office at info@mathcounts.org.

Step-by-step solutions to these problems are available to registered Competition Series Coaches online or in the print version of the School Handbook available for purchase at mathcountsstore.com.

Competition Series
COACHES!

FIND THE 2021–2022 SCHOOL HANDBOOK WITH SOLUTIONS AT

mathcounts.org/coaches

Math Video Challenge
ADVISORS!

FIND THE 2021–2022 SCHOOL HANDBOOK AT

mathcounts.org/mvcteams

HIGHLIGHTED RESOURCES

For Competition Coaches & Students

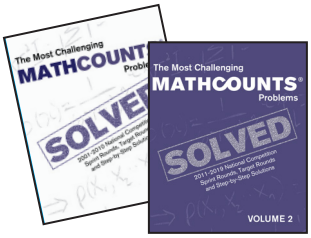
12-Month OPLET Subscription

Online database of over 13,000 problems and over 5,000 step-by-step solutions. Create personalized quizzes, flash cards, worksheets and more!

Save $25 when you buy your subscription by Oct. 15, 2021
Renewers: use code PHOENIX21
First-Time Subscribers: use code FEATHER21

mathcounts.org/myoplet

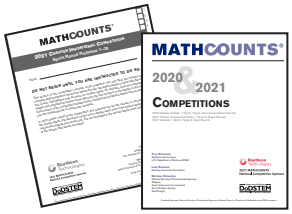
Most Challenging MATHCOUNTS Problems Solved, Vol. 1 & Vol. 2



Each volume has over 300 national-level Sprint and Target Round problems, plus detailed step-by-step solutions. A fantastic advanced-level practice resource!

mathcounts.org/store

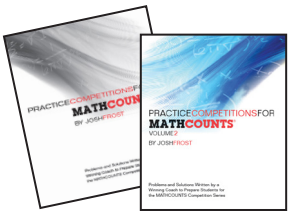
Past Competitions



Last year's school, chapter and state MATHCOUNTS competitions are free online! Other years and levels of competitions can be purchased.

mathcounts.org/pastcompetition
mathcounts.org/store

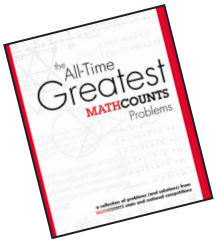
Practice Competitions for MATHCOUNTS, Vol. I & Vol. II



Each volume has 4 complete mock competitions and solutions written by repeat national-level coach Josh Frost.

mathcounts.org/store

All-Time Greatest MATHCOUNTS Problems



A collection of some of the most creative and interesting MATHCOUNTS problems.

mathcounts.org/store

MATHCOUNTS Trainer, by Art of Problem Solving

Train your Mathletes with this Countdown Round-style game featuring realtime leaderboards and lots of past MATHCOUNTS problems. Play online or get the iOS app!

mathcounts.org/trainer

GET MORE AT MATHCOUNTS.ORG/COACHES

= free resource!

For Math Video Challenge Advisors & Students

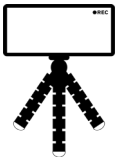
Copyrighted Materials Guidelines

Learn how to use video clips, music, images and sound effects while following the contest rules.

Outside Help Guidelines

Distinguish between appropriate advising and inappropriate outside help.

Technical Guidelines



Requirements and recommendations for filming and editing.

Judging Guidelines

Learn more about what judges are looking for as they evaluate a video.

Animated Video Spotlight & Guidelines

Tips, recommended programs and examples for creating an animated video.

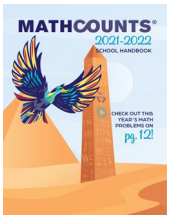
Video Spotlight

Collection of videos that received high marks from judges in at least 1 category.
Each example includes a detailed explanation of what the judges liked.

ALL AT MATHCOUNTS.ORG/MVCTEAMS

For Everyone

MATHCOUNTS School Handbook Remix



All 250 problems, answers & solutions in the Handbook organized by math topic.

mathcounts.org/handbook

MATHCOUNTS Minis

A fun monthly video series featuring Richard Rusczyk from AoPS. Each video walks through problem-solving strategies for a math skill common in MATHCOUNTS problems.

mathcounts.org/minis

Problem of the Week

A new, multi-step problem every Monday! Each problem coincides with a timely event, holiday or season. Step-by-step solutions for each problem are shared the following week.

mathcounts.org/potw

Coming Soon!

We've teamed up with Brilliant to provide coaches and club leaders free access to interactive lessons and tools through Brilliant for Educators. Stay tuned for more information!

2021–2022 PROGRAM INFO

MATHCOUNTS® COMPETITION SERIES

An exciting math contest for students in grades 6-8. Students practice with coaches in the fall and then participate in 4 competition levels:

- **School:** run by coaches (like you!)
- **Chapter** & **State**: run by 600+ *amazing* volunteer coordinators
- **National**: run by the national office

Wondering where to start?

GUIDE FOR NEW COACHES:
mathcounts.org/newcoach

COACH RESOURCES:
mathcounts.org/coaches

MATHVIDEO CHALLENGE

A free, fun math and technology contest for students in grades 6-8. Students work in teams of 4 to create a video that explains the solution to a MATHCOUNTS handbook problem in a real-world scenario.

Wondering where to start?

GUIDE FOR NEW TEAM ADVISORS:
mathcounts.org/newadvisor

VIDEO TEAM RESOURCES:
mathcounts.org/mvcteams

THE NATIONAL MATHCLUB

Try the National Math Club, a free program for students in grades 6-8 to explore math with fun, collaborative activities.

Get even more resources!

REGISTER:
mathcounts.org/register

Critical Dates*

2021

M Aug. 16-Dec. 31 — **Registration and kit fulfillment open for all 3 MATHCOUNTS programs**

The fastest way to register for any MATHCOUNTS program is at **mathcounts.org/register**.

Competition Series coaches and NSCs receive a School Competition Kit, which includes a hard copy of this Handbook. Kits are shipped on an ongoing basis from mid-Aug. to late Jan.

Math Video Challenge advisors receive a Producers Kit, which includes a smartphone tripod and a hard copy of this Handbook. Kits are shipped on an ongoing basis from mid-Aug. to late Feb.

National Math Club leaders receive a Club Kit, which includes game supplies and a Club Leader Guide. Kits are shipped on an ongoing basis from mid-Aug. to late May.

Nov. 1 — The **2022 School Competition** will be available at mathcounts.org/coaches.

Nov. 12 — **Early Bird Registration Deadline:** $30/student for schools and $60/non-school competitor (NSC).

Dec. 17 — **Regular Registration Deadline:** $35/student for schools and $65/non-school competitor (NSC). MATHCOUNTS cannot guarantee participation for schools that register late.

Critical Dates*

2022

Jan. 1	**Registration and kit fulfillment continue** for all 3 MATHCOUNTS programs. Late Competition Series registrations may be accepted at the discretion of MATHCOUNTS and local coordinators. Math Video Challenge registration ends Mar. 11 and National Math Club registration ends May 31, but we recommend registering for both ASAP.
early Jan.	If you haven't received details about your upcoming competition, **contact your local or state coordinator**. Get coordinator contact info at mathcounts.org/findmycoordinator.
Jan. 11	**Recommended registration deadline**. Those who finish team registration, including parent/guardian permission, by 11:59pm ET will be entered into **Prize Drawing 1**.
late Jan.	If you have not received your School Competition Kit, **contact the MATHCOUNTS national office** at info@mathcounts.org.
Feb. 1-28	**2022 Chapter Competitions***
Feb. 11	**Prize Drawing 2** for for teams that have videos submitted and approved by their team advisor by 11:59pm ET.
late Feb.	If you have not received your Producers Kit, **contact the MATHCOUNTS national office** at info@mathcounts.org.
Mar. 1-31	**2022 State Competitions***
Mar. 11	**Final registration and submission deadline** at 11:59pm ET for prizes and advancement. Any offline permission forms must be submitted by 5:00pm ET.
Mar. 18	**Quarterfinalist Videos announced**
Mar. 25	**12 Semifinalist Videos and 6 Judges' Choice Winners announced**
Apr. 1	**Gold & Silver application deadline to qualify for prizes and drawings** Learn how to apply at mathcounts.org/clubleaders.
Apr. 4	**4 Finalist Videos Announced**
Apr. 8	**Grand Prize, Gold & Silver drawing winners announced**
May 8-9	**2022 Raytheon Technologies MATHCOUNTS National Competition** in Washington, DC, including the Math Video Challenge Finals.*
Jun. 1	**Final Gold & Silver application deadline to receive certificates**

*MATHCOUNTS is preparing for in-person events at all levels of competition for the 2021-2022 season. At the same time, we will closely monitor the impact of COVID-19 on school districts and volunteers. Should safety guidelines or government restrictions necessitate, the MATHCOUNTS Competition Series would be conducted in a similar manner to the successful online 2020-2021 Competition Series.

THIS YEAR'S HANDBOOK PROBLEMS

ALGEBRAIC EXPRESSIONS + EQUATIONS *pages 7-8*	**NUMBER THEORY** *pages 12-14*	**PROPORTIONAL REASONING** *pages 23-24*
COORDINATE GEOMETRY *pages 8-9*	**PERCENTS + FRACTIONS** *pages 14-15*	**SEQUENCES, SERIES + PATTERNS** *pages 24-25*
GENERAL MATH *page 9*	**PLANE GEOMETRY** *pages 15-17*	**SOLID GEOMETRY** *pages 25-26*
LOGIC *page 10*	**PROBABILITY, COUNTING + COMBINATORICS*** *pages 18-21*	**STATISTICS + DATA** *pages 26-28*
MEASUREMENT *pages 11-12*	**PROBLEM SOLVING (MISC.)** *page 22*	**Refer to page 18 for a mini lesson on the Stars and Bars technique for #21-30.*

Competition Series
COACHES!

FIND THE 2021–2022 SCHOOL HANDBOOK WITH SOLUTIONS AT

mathcounts.org/coaches

Math Video Challenge
ADVISORS!

FIND THE 2021–2022 SCHOOL HANDBOOK AT

mathcounts.org/mvcteams

Problem #

35. How many positive integers x satisfy the inequality $x^2 + x < 100$?

36. Let $a \# b = a^2 - b^2$. What is the value of $(3 \# 2) - (4 \# 3)$?

37. If $2x + 7 = 17$, what is the value of $8x + 25$?

48. It is known that $a = \frac{b}{c}$ and $b = d + e$. If $a = 3$, $d = 7$ and $e = 11$, what is the value of c?

54. What value of x satisfies the equation $x + \frac{x}{2x + 4x} = 2$? Express your answer as a common fraction.

58. If $36 + 64 + 45 + 55 + 56 + 44 + 67 + 33 + 78 + N = 500$, what is the value of N?

65. What value of x satisfies the equation $2x + \frac{1}{5} = \frac{2}{3}$? Express your answer as a common fraction.

71. Let $x \diamond y = x + y + xy$. If $x \diamond 3 = 15$, what is the value of x?

73. Based on the system of equations given, what is the value of M + A + T + H?

$$\begin{aligned} M + A &= T \\ A + T &= H \\ T + H &= M \\ H + M &= A \end{aligned}$$

75. Currently, Abdi is 24 years older than Eric. In 3 years, Abdi will be 3 times as old as Eric. What is the sum of their current ages?

90. The expression $\frac{(a^3b^5c^2)^3(a^5b^2c^9)^2}{a^4b^9c^4}$ can be written as $(a^xb^yc^z)^5$. What is the value of $x + y + z$?

99. What is the greatest possible value of x that satisfies the inequality $\frac{1}{x} + \frac{1}{2} \geq 5$? Express your answer as a common fraction.

102. What is the sum of all solutions w to the equation $|4 - 5w| = 2$? Express your answer as a common fraction.

106. If $\frac{x}{x - 1} = 2$, what is the value of x?

108. Marika lives 20 miles from work. If she drives to work at 50 mi/h, how fast will she need to drive home to average 60 mi/h for the round trip? Express your answer to the nearest whole number.

114. What is the least value of x that satisfies the equation $x + \frac{1}{x} = \frac{17}{4}$? Express your answer as a common fraction.

132. How many different values of x satisfy the equation $|x^2 - 9| = |2x + 6|$?

134. What is the sum of all positive integers t that satisfy the inequality $4t + 5 < 100$?

149. Gene has \$52 in his wallet. He has some \$1 bills, some \$5 bills, and some \$10 bills. If he has 3 times as many \$1 as \$5 and twice as many \$5 as \$10 bills, how many \$10 bills does he have?

Problem #

150. According to a study in the *European Journal of Clinical Nutrition*, a woman's projected height (PH) in centimeters is related to her hand length (HL) in centimeters by the following formula: PH = 80.400 + 5.122 × HL − 0.195 × age in years. Given that Kiera is 36 years old and that her hand length is 14.96 cm, what is her projected height? Express your answer to the nearest whole number.

155. On Monday, Jamal biked 4 miles from home to school at an average speed of 10 mi/h. After school, he noticed that his bike had a flat tire, so he walked home. If he walked at a speed of 3 mi/h, what was his average speed on the trip to school and back home? Express your answer as a decimal to the nearest tenth.

180. The formula for converting temperature in degrees Celsius (C) to degrees Fahrenheit (F) is $F = \frac{9}{5}C + 32$. If Cai misreads the Celsius temperature by 2°, by how many degrees Fahrenheit will she be off? Express your answer as a decimal to the nearest tenth.

184. If $\frac{x-1}{x+1} = 3$, what is the value of x?

219. What is the sum of the values of x such that $\frac{1}{x} + \frac{1}{x+1} = \frac{1}{x+2}$?

239. What is the smallest positive three-digit integer n for which each of the expressions $\frac{3n+4}{5}$, $\frac{4n+5}{3}$ and $\frac{5n+3}{4}$ represents an integer?

241. Suppose $(a, b) \blacklozenge\blacklozenge (c, d) = (ac - bd, ad + bc)$ for real numbers a, b, c and d. Given that $(a, b) \blacklozenge\blacklozenge (a, b) = (-1, 0)$, what is the value of $a^4 + b^4$?

249. Art opened a savings account that earned compound interest, compounded monthly. After the initial deposit, Art made no deposits and no withdrawals. At the end of 8 months his account balance was \$499.54, and at the end of 18 months the balance was \$525.09. How much money did Art initially deposit in this account? Express your answer to the nearest whole dollar.

Cg Coordinate Geometry

Problem #

20. A circle of radius 17 units has its center in the second quadrant. The circle intersects the y-axis at (0, 0) and $(0, 17\sqrt{2})$. What is the area of the region of the circle that lies in the third quadrant? Express your answer as a decimal to the nearest tenth.

82. A triangle has vertices A(−1, 2), B(5, 7) and C(−1, −7). Point D(x, y) is on side BC, and the area of triangle ACD is half the area of triangle ABC. What is the value of $x + y$?

92. What is the distance in space between the points (1, 2, 3) and (−7, 6, 4)?

119. The graph of the line $y = 6x - 5$ intersects the graph of the parabola $y = x^2$ at two points (x, y). What is the distance between those two points? Express your answer in simplest radical form.

Problem #

163. Points A, B, C and D have coordinates A(5, 7), B(7, 1), C(17, m) and D(−1, 3). If line segments AB and CD are perpendicular, what is the value of m?

198. The graph of $4x - y = r$ intersects the graph of $5x + 2ty = -30$ at the point (−2, −5). What is the value of $t - r$?

218. A *lattice point* can be defined as a point in the xy-plane for which both coordinates are integers. How many lattice points does the graph of $x^2 + y^2 = 25$ pass through?

Gm General Math

Problem #

33. There are 10 apple trees at the orchard. On each tree are 100 apples. A total of 30 apples are eaten by worms, and 100 apples are pressed for cider. How many apples are left?

34. If the addition problem shown here is correct, what is the value of A?

$$\begin{array}{r} A \\ +\ A \\ \hline 16 \end{array}$$

39. What is the value of 13 + 17 + 23 + 27 + 33 + 37 + 43 + 47 + 53 + 57?

43. The figure shows the costs of several items sold at a candy store. Piri buys 3 lollipops, 2 peppermints, 4 butterscotches and 8 gumballs from this store and pays 12 cents tax. If Piri pays with a $5 bill, how much change should he get back?

Candy	Price
Lollipop	25¢
Taffy	10¢
Gumball	10¢
Butterscotch	5¢
Peppermint	5¢

49. What is the value of the expression shown here?

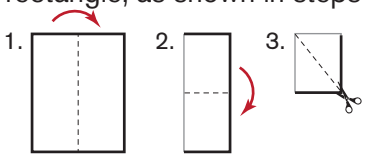
$$\begin{array}{l} 789+789+789+789+ \\ 789+789+789+789+ \\ 789+789+789+789+ \\ 789+789+789+789+ \\ 789+789+789+789 \end{array}$$

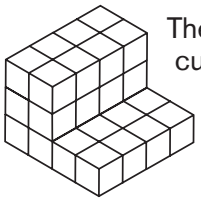
79. What is the absolute difference between 0.40 and $\frac{7}{5}$?

141. Annie has one quarter, two dimes and three nickels. Oliver has two quarters, one dime and one nickel. How much money do Oliver and Annie have, combined?

Problem #

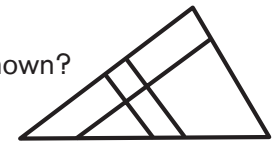
61. Sadako has a 6-inch by 8-inch rectangle of paper. She folds it in half from left to right, and then in half from top to bottom, so that the folds are on the top and left of the resulting rectangle, as shown in steps 1 and 2. Then she cuts the paper along a straight diagonal line from the bottom right corner to the top left corner, as shown in step 3. When she unfolds the paper, how many separate pieces are there?



70. 

The figure shows a solid composed of 32 unit cubes. What is the least number of unit cubes that can be added to this figure to create a solid that is a cube?

100. How many more quadrilaterals than triangles are in the figure shown?



103. What is the sum of all positive integers $n \leq 20$ that satisfy the condition that if n is odd, then n is a multiple of 5?

142. At 22 years old, Clara is the oldest of her parents' eight daughters, all of whom share the same birthday. Each of Clara's younger sisters is exactly two years younger than her next oldest sister. What is the sum of the ages of all eight daughters?

192. In this conversation shown between Anne and Diana, if Diana is correct, what is the greatest possible value of X?

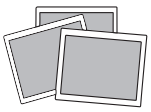
> ***Anne:*** When I divide 100 by my favorite number, the remainder is 2.
> ***Diana:*** Is your number less than X?
> ***Anne:*** Yes.
> ***Diana:*** Then I know what your favorite number is.

232. How many rectangles of any size appear in the figure shown?

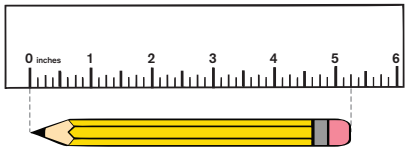
234. There are 50 eighth-grade students at Gauss Middle School. If 36 of the eighth graders practice non-Euclidean geometry, 29 practice adding series, and 3 students practice neither, how many eighth-grade students practice both?

Problem #

16. Adele draws a circle with radius 10 inches. Bernie draws a circle that has four times the area of Adele's circle. What is the diameter of Bernie's circle?

31.  Cam took three photos and printed one 4-inch by 6-inch print of each photo. What is the total area of the photos she printed, in square inches?

41. Betty measures a pencil with a ruler that measures inches, as shown. How many inches long is the pencil? Express your answer as a mixed number.



46.  Starting at his house, Lee bikes 3 miles west, then 2 miles north, then 3 miles west, then 8 miles south, then 2 miles west and finally in a straight line back to his house. How many miles did Lee bike in total?

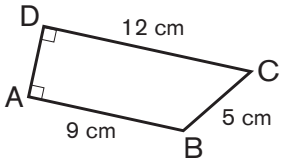
51. Bob is exactly 34 inches taller than his son, Tim. If Tim is 3 feet, 2 inches tall, how tall is Bob, in feet?

60. What is the area of the triangle with vertices at A(2, 2), B(4, 5) and C(7, 5)? Express your answer as a decimal to the nearest tenth.

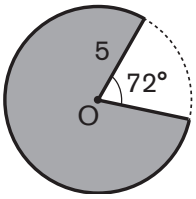
77. What is the area of a square that has a perimeter of 10 inches? Express your answer as a decimal to the nearest hundredth.

91. A circle has circumference $4\pi^2$ units. What is the circle's area? Express your answer in terms of π.

104. In trapezoid ABCD, shown here, bases AB and CD have lengths 9 cm and 12 cm, respectively, and side AD is perpendicular to each base. If BC = 5 cm, what is the area of trapezoid ABCD?



116. Circle O has radius 5 meters. When a 72° sector is removed from circle O, as shown, what is the area of the shaded region that remains? Express your answer in terms of π.



159. Assume that Earth is a perfect sphere with radius 4000 mi. In miles, what is the distance in a straight line from a point on the equator through the planet to the South Pole? Express your answer in simplest radical form.

170. What is the length of a rectangle that has width 5.6 meters and area 70 m^2? Express your answer as a decimal to the nearest tenth.

Problem #

176. Mr. Brown has 20 volumes of an old-fashioned encyclopedia set filling a 1-meter long bookshelf. Together, the front and back covers of each volume are 3.981 mm thick. Each page is 0.0310 mm thick. How many pages are in the entire encyclopedia set? Express your answer to the nearest hundred.

186. The area of an isosceles triangle is 40 in^2. The length of the triangle's base is 8 inches. What is the perimeter of the triangle? Express your answer as a decimal to the nearest hundredth.

193. Justin said he traveled 8 miles, when, in fact, he traveled 8 nautical miles. If a nautical mile is 1.15078 miles, how many miles did Justin travel? Express your answer as a decimal to the nearest hundredth.

197. In square millimeters, what is the area of a triangle with side lengths 115 mm, 9.2 cm and 0.069 meters?

200. The great pyramid of Giza is a pyramid with a square base that is 230 meters on each side, and it is 145 meters tall. It is built mostly of limestone that has a density of 2900 kg/m^3. Assume that the entire pyramid is solid. What is the pyramid's mass, in kilograms? Express your answer in scientific notation to three significant digits.

242. Convex quadrilateral ABCD has AB = 4, BC = 5, AC = 5, AD = 3 and CD = 4. What is the area of quadrilateral ABCD? Express your answer as a decimal to the nearest tenth.

Nt Number Theory

Problem #

40. What is the sum of all positive integer factors of 20?

45. What is the sum of all the one-digit prime numbers?

52. What is the closest integer to 12.2 × 7.8?

59. At Turtle Elementary, the kindergarten teachers lined up their students by 6s and there were none left over. They lined them up again by 8s and there were none left over. They tried to line them up again by 10s, but there were 2 left over. What is the least number of students that could be in kindergarten at Turtle Elementary?

62. What is the value of $\left(1\frac{1}{3}\right)^2 + \left(1\frac{1}{3}\right)^{-2}$? Express your answer as a common fraction.

63. What is the sum of the distinct prime factors of 6300?

66. Let $a \Uparrow b = a^2 + b^2$. What is the value of $[(3 \Uparrow 1) \Uparrow 2] - [3 \Uparrow (1 \Uparrow 2)]$?

81. In the equation $376 - M = X$, where X is a two-digit positive integer, what is the greatest possible value of M?

85. What is the value of the expression $\dfrac{2^{24} - 2^{22}}{2^{23} - 2^{21}}$?

86. A and B are two-digit positive integers, and the product $A \times B$ is a four-digit integer. What is the least possible value of A?

Problem #

97. If a number n is divided by 4, the remainder is 1. What is the remainder when $5n$ is divided by 4?

107. What is the value of the expression $\dfrac{22^2 - 17^2}{20^2 - 19^2}$?

109. What is the value of 23_5 in base ten?

112. If $(\sqrt{2})^n = \sqrt[16]{16}$, what is the value of n? Express your answer as a common fraction.

118. How many integers n satisfy $12 < \sqrt{n} < 13$?

120. What is the units digit of 17^{21}?

124. What is the greatest three-digit number that is divisible by 11 and contains the digit 5 at least once?

128. The five-digit number 5A816 is divisible by 24. What is the sum of all possible digits A?

130. How many positive integers less than or equal to 1000 are multiples of 3 or multiples of 10?

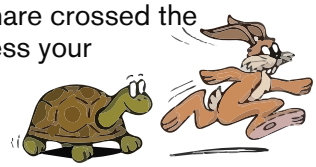
137. What is the greatest prime p for which $10 < \sqrt{p} < 11$?

171. The positive even integer n has a square root that is greater than 7.9 and a cube root that is less than 4.3. The sum of the digits of n is 14, and the product of its digits is greater than 35. If n is not a multiple of 3, what is the value of n?

172. The sum of the squares of two consecutive positive odd integers is 130. What is the sum of the two numbers?

173. What is the sum of the positive multiples of 3 that are less than 1000?

183. A tortoise challenged a hare to a 5-mile race. The tortoise ran the entire race at a steady speed of 0.63 mi/h. The hare ran the entire race at a steady speed of 36 mi/h. The hare crossed the finish line at noon. At what time did the tortoise cross the finish line? Express your answer in hours and minutes to the nearest minute.



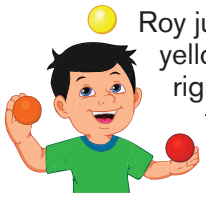
199. The sum of four positive integers is 7. What is the least possible sum of their reciprocals? Express your answer as a mixed number.

202. Let $\langle A \rangle$ equal the greatest multiple of 100 less than A. What is the value of $\langle 98^2 \rangle$?

204. If n is an integer and $10 < 6^n < 10{,}000$, what is the sum of all possible values of n?

207. What is the greatest prime divisor of $13! - 12! + 11!$?

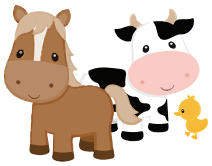
208. The three-digit number ABC is 675 more than the three-digit number DEF. If the letters A through F represent distinct positive digits other than 6, 7 or 5, what is the value of ABC?

209. 

Roy juggles three balls—one red, one orange and one yellow. He starts by throwing the yellow ball into the air while the red ball is in his left hand and the orange ball is in his right hand. Every second, the yellow ball takes the place of the red ball, the red ball takes the place of the orange ball, and the orange ball takes the place of the yellow ball. How many seconds after starting will he take the red ball in his right hand for the fifth time?

Problem #

215. What is the greatest integer value of n such that 3^n is a factor of 99!?

217. 

A duck quacks every 3 minutes, a cow moos every 7 minutes, and a horse neighs every 12 minutes. At noon, the animals all sound off together. What is the next time that all three sound off together again?

225. Suppose that N has an odd number of positive integer divisors, N is even and $N < 1000$. What is the greatest possible value of N?

226. The ratio $3{,}579{,}545.\overline{45}:5{,}000{,}000$ can be expressed as a common fraction $\frac{m}{n}$. What is the value of $m + n$?

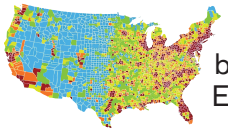
246. How many divisors of 11! are both odd and a perfect square?

Pf Percents + Fractions

Problem #

44. What number is $\frac{1}{3}$ of the way from $\frac{1}{12}$ to $\frac{5}{12}$? Express your answer as a common fraction.

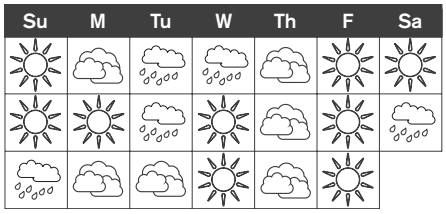
67. What percent of 3 is 30?

83. 

In 2019, the US population was 330 million while the world population was 7.7 billion. In 2019, what percentage of the world population lived in the United States? Express your answer to the nearest tenth of a percent.

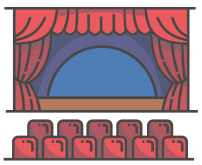
96. What is $\frac{1}{10}$ of $\frac{1}{5}$ of 80? Express your answer as a decimal to the nearest tenth.

143. For a school project, Yun records the weather at his house every day for a 20-day period in September. On what percent of the days did it rain? Express your answer to the nearest percent.



151. Delilah brought a red ball, a green top and a blue yo-yo to her playdate with Emily and Francesca. Emily brought an orange ball, a red Hula-Hoop and two blue marbles. Francesca has a green ball, a yellow top and a blue jump rope. What percent of the toys brought to the playdate were blue?

158. A theater holds 290 people. If 261 people are in attendance, what percent of the theater is empty?



161. If 125% of p is 25, what is the value of p?

166. 

A game console is marked as 15% off its original price. Lou buys it and pays \$250.75. By how much was the original price reduced?

Problem #

167. The number of students in the Chess Club is 40% of the number of students in the Math Club, which is 30% of the number of students in the Drama Club. The number of students in the Chess Club is what percent of the number of students in the Drama Club?

168. The table shows all five lunch options on Friday at a middle school and the number of students who ordered each. What percent of the lunch orders on Friday were for spaghetti?

Student Lunch Data

Lunch	Orders
Pizza	132
Nuggets	112
Burger	88
Spaghetti	44
Salad	24

181. The positive number a is 160% of another number, b. What percent of $25a$ is $16b$?

189. Kayla has one gallon of orange juice. How many quarts of pineapple juice will she need to add to make a mixture that is 80% orange juice?

190. What is the 50th digit after the decimal point in the expansion of $\frac{991}{37}$?

194. The clothing inventory at Cool Cat Clothes is given in the chart shown. What percent of Cool Cat's inventory do jackets comprise?

Item	Qty
Hats	46
Jackets	65
Pants	74
Shirts	87
Vests	53

248. A positive number n is 25% of $\frac{6}{5}$ of its square. What is the value of n? Express your answer as a decimal to the nearest tenth.

Pg Plane Geometry

Problem #

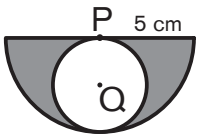
11. What is the circumference of a circle that has an area of 289π mm^2? Express your answer in terms of π.

12. The area of a circle with a diameter of $2\sqrt{13}$ feet equals $b\pi$ ft^2. What is the value of b?

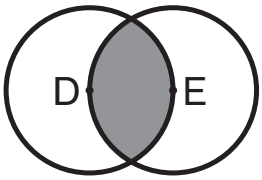
13. If the diameter of a circle is multiplied by four, the area of the new circle is how many times that of the original circle?

14. The circumference of a certain circle with radius r cm is equal to the perimeter of a certain square with side length s cm. What is the ratio of r to s? Express your answer as a decimal to the nearest tenth.

15. Semicircle P has radius 5 cm, and circle Q is the largest possible circle that can be inscribed in semicircle P. What is the combined area of the shaded regions that are in the interior of semicircle P but exterior to circle Q? Express your answer as a common fraction in terms of π.



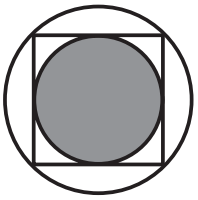
17.

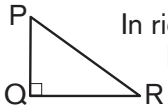


Circle D intersects the center of circle E, and circle E intersects the center of circle D. The radius of each circle is 6 cm. The area of the shaded region where the circles overlap can be expressed in simplest radical form as $a\pi + b\sqrt{c}$ cm^2. What is the value of $a + b + c$?

Problem #

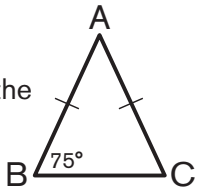
18. The figure shows a circle inscribed in a square that is inscribed in a circle. If the larger circle has area 256π mm^2, what is the radius of the smaller, shaded circle? Express your answer in simplest radical form.



68. 

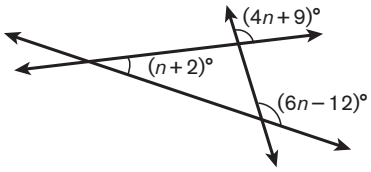
In right triangle PQR, $m\angle P = 60$ degrees, $m\angle R = 30$ degrees and PR = 20 cm. What is the length of side QR? Express your answer in simplest radical form.

80. In isosceles triangle ABC, shown here, AB = AC, and the $m\angle B = 75$ degrees. What is the degree measure of ∠A?

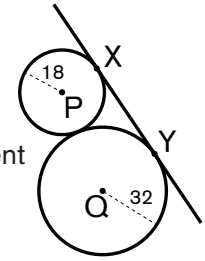


89. The sum of the interior angle measures of a regular polygon is 900 degrees. How many sides does the polygon have?

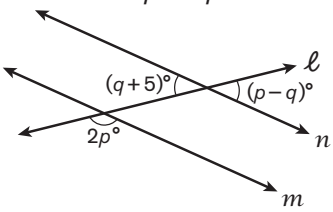
93. In the figure shown, what is the value of n?

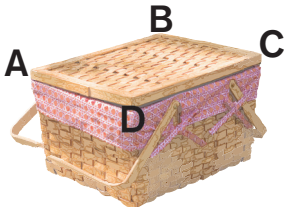


121. Circles P and Q are externally tangent to each other and have radii 18 meters and 32 meters, respectively. As shown, circles P and Q are externally tangent to a line segment at points X and Y, respectively. What is XY?



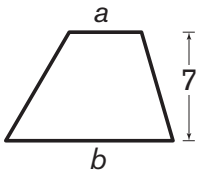
123. If line ℓ intersects parallel lines m and n as shown, what is the value of $p + q$?



133. 

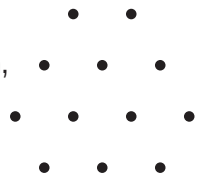
An ant walks clockwise along the rectangular rim of the picnic basket shown from point A to B to C to D, a distance of 28 ¼ inches. A ladybug walks clockwise along the rim of the same picnic basket from point B to C to D to A, a distance of 25 inches. What is the perimeter of the picnic basket's rim? Express your answer as a mixed number.

138. A rectangle is inscribed in a circle. If one side of the rectangle is 6 cm and the area of the circle is 25π cm^2, what is the area of the rectangle?

139. 

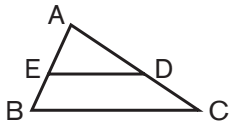
The trapezoid shown has bases of lengths a units and b units and height 7 units, where a and b are positive integers and $a < b$. If the trapezoid has area 49 units^2, how many possible values are there for a?

140. Twelve points are arranged as shown on an equilateral triangular lattice grid, with the lattice points spaced 1 cm apart. If all the points are connected to form a simple polygon, by using line segments of 1 cm each, what is the maximum possible area, in square centimeters, that can be enclosed by the polygon? Express your answer in simplest radical form.

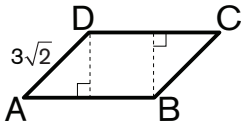


Problem #

179. If the area of triangle ABC is 4 times that of triangle ADE, what is the ratio of the area of triangle ABC to that of trapezoid BCDE? Express your answer as a common fraction.



188. Parallelogram ABCD, shown here, is made of two isosceles right triangles and a square. If AD = $3\sqrt{2}$ units, what is the area of ABCD?



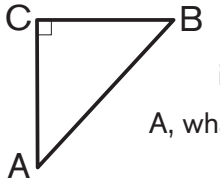
195. A circle is inscribed in an equilateral triangle of side 12 meters. What is the diameter of the circle? Express your answer as a decimal to the nearest tenth.

205. For a regular polygon, the complement of an exterior angle is $\frac{1}{4}$ of the supplement. How many sides does the polygon have?

213. A circle is drawn inside triangle ABC so that it is tangent to all three sides of the triangle. Triangle ABC has perimeter 24 cm and area 8 cm^2. What is the radius of the circle? Express your answer as a common fraction.

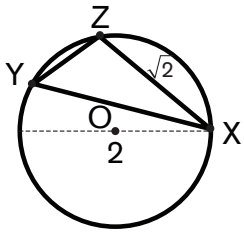
223. In triangle ABC, AB = 5 and BC = 17. How many different possible integer lengths can side AC have?

227.

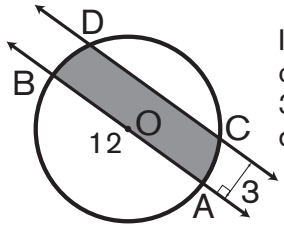


In right triangle ABC, the ratio $\frac{BC}{AB}$ is called the sine of angle A, and the ratio $\frac{AC}{AB}$ is called the sine of angle B. If the sine of angle B is the same as the sine of angle A, what is the degree measure of angle A?

230. Triangle XYZ is inscribed in circle O as shown. If circle O has a diameter of 2 units and XZ = $\sqrt{2}$ units, what is the degree measure of $\angle XYZ$?



235.



In the figure shown, lines AB and CD are parallel, line AB passes through the center of circle O, AB = 12 cm, and the distance between lines AB and CD is 3 cm. What is the area of the shaded region? Express your answer as a decimal to the nearest tenth.

Stars and Bars (for problems 21–30)

Stars and bars, also known as balls and urns or sticks and stones, is a technique for finding the number of ways to distribute indistinguishable items (stars, balls, stones) among distinguishable containers (like urns) or distinguishable groups (separated by bars and sticks). The number of separators used is always one less than the number of containers or groups. Here are two examples of the stars and bars technique.

Here are arrangements of stars and bars that show four ways to distribute 8 pennies among 3 piggy banks.

	A	B	C
* * * * * * * * | |	8	0	0
* * * | * * | * * *	3	2	3
* * * | | * * * * *	3	0	5
| * * * * * * * * |	0	8	0

In how many ways can 8 pennies be distributed among 3 piggy banks?

The number of arrangements of 8 stars and 3 − 1 = 2 bars can be computed from the formula for permutations of items of different types. There are 8 + 2 = 10 items, 8 stars and 2 bars, and the formula tells us that the number of permutations is $\frac{(8+2)!}{8!2!} = \frac{(10 \times 9)8!}{8!(2 \times 1)} = 5 \times 9 = 45.$

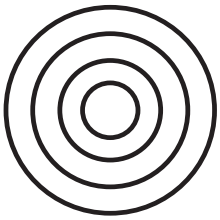
The formula for combinations can also be applied here, viewing the problem as choosing the locations for the 2 bars among the 8 + 2 = 10 spots for stars or bars, leading to exactly the same computation.

$$\binom{s+b}{b} = \frac{(s+b)!}{s!b!}$$

In how many ways can a group of 10 balloons be made using red, orange, yellow, blue and purple balloons if there must be at least one balloon of each color?

We need only count the number of ways 5 of the balloons are colored since 5 have predetermined colors. The number of ways to choose 5 − 1 = 4 of the 5 + 4 = 9 items to be bars is $\binom{9}{4} = \frac{9!}{4!(9-4)!} = \frac{(9 \times 8 \times 7 \times 6)5!}{(4 \times 3 \times 2 \times 1)5!} = 9 \times 7 \times 2 = 126.$

Problem #

19. 

Tomas shoots an arrow that lands in a random location on the target shown. The radius of the center circle is 1 foot, and the radius of each successively larger circle is 1 foot greater than that of the previous circle. What is the probability that Tomas's arrow will land in the center circle? Express your answer as a common fraction.

21. In how many ways can 9 yellow marbles be divided among 4 distinguishable cups?

22. A tray contains a dozen each of 3 kinds of cookies. How many different assortments of 7 cookies can Devon select from the tray?

23. Baca's Bakery sells chocolate, vanilla, pumpkin and carrot cake muffins. How many different assortments of 10 muffins can Baca make?

24. How many different assortments of pennies, nickels, dimes and quarters can Ashley's coin holder contain if it has 15 coins total?

25. 

At the flower shop, Maggie is making a bouquet from asters, dahlias, irises, roses and chrysanthemums. How many ways can Maggie choose 9 flowers for her bouquet if it should contain at least one of each type of flower?

Problem #

26. Carla selects 5 fruit-flavored candies from a bowl containing 6 apple, 5 banana and 4 cherry candies. How many possible combinations of candies can Carla select?

27. Lynn has 3 cats and a row of 4 cat beds. Each cat bed can hold one or two cats, and each cat is in a bed. Listing the number of cats in each bed from left to right, how many unique sequences are there?

28. Marvin randomly places 7 Scuba Steve action figures and 6 Diving Dan action figures in 4 distinguishable boxes. If each box must have at least one of each type of action figure, how many ways can Marvin do this?

29. Ms. Grow has 10 identical MATHCOUNTS pencils to distribute to 5 Mathletes. Two brothers, Minhtet and Linnhtet, must get the same number of pencils. If not every Mathlete is required to get a pencil, in how many ways can Ms. Grow distribute the pencils?

30. How many four-digit integers are there with a digit sum of 31?

38. Willy lost a chocolate drop somewhere on the floor of his candy factory. The factory floor measures 140 yards by 120 yards. Willy searches a rectangular section of the factory floor that is 70 yards by 30 yards. If the chocolate drop was equally likely to fall anywhere on the factory floor, what is the probability that the chocolate drop is in the section Willy searches? Express your answer as a common fraction.

55. What is the probability that a randomly chosen positive divisor of 80 is also a divisor of 96? Express your answer as a common fraction.

76.

Lisa has three days to see six movies. She wants to see *Live Soft*, *Sits with Coyotes, Thawed, The So-So Five, Thallium Woman* and *East by Southeast.* How many ways can Lisa arrange her schedule for Friday, Saturday and Sunday so that she sees two different movies each day, if the order of movies on any given day does not matter?

87. A pair of six-sided dice are rolled. What is the probability that the product of the numbers shown is a multiple of 3? Express your answer as a common fraction.

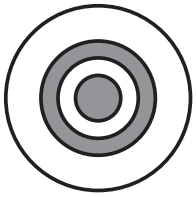
94.

A fair coin is flipped 10 times. What is the probability that neither heads nor tails shows up twice in a row? Express your answer as a common fraction.

98. The odds that Macy will come in first in the 200-meter hurdles are said to be 5 to 12. This quantity represents the ratio of the probability that she will come in first to the probability that she will not come in first. What is the probability that someone else will come in first in the race? Express your answer as a common fraction.

105. How many ordered triples (a, b, c) of nonnegative integers exist for which $a + b + c = 4$?

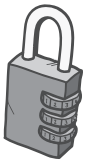
110.

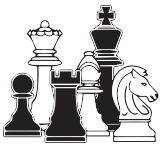


Darts land randomly on a dartboard consisting of concentric circles of radii 3, 5, 7 and 11 inches. The probability that a dart lands in the shaded ring is how many times the probability that it lands in the shaded circle? Express your answer as a mixed number.

Problem #

115. Ashwin remembers that the combination for his locker consists of the numbers 7, 13 and 3. However, he can't remember the order in which the three numbers are to be entered. If he randomly picks one of the possible orderings, what is the probability that Ashwin will open the lock on his first try? Express your answer as a common fraction.



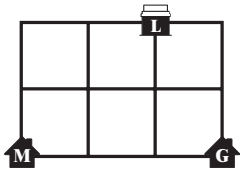
117. 

Matilda has a box containing a set of standard chess pieces, 16 black and 16 white. She selects 3 pieces at random, without replacement. What is the probability that all 3 chess pieces are the same color? Express your answer as a common fraction.

122. Joe has a bag filled with 17 yellow marbles, 9 blue marbles and 9 pink marbles. He randomly selects a marble from the bag. What is the probability that Joe will select a blue marble? Express your answer as a common fraction.

127. Ursula wants to make a bouquet of a dozen roses. The flower shop has an ample supply of red and white roses. How many different combinations of roses are there for Ursula to create this bouquet?

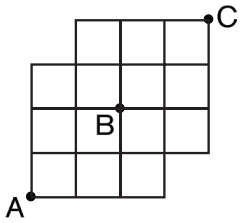
129. This map of six square blocks shows the location of Mae's house (M), her grandfather's house (G) and the library (L). How many different routes can Mae take to walk from her house to the library to get a book and then to her grandfather's house for lunch, assuming she does not walk more than a total of 7 blocks?



135. A fair coin is flipped 8 times. What is the probability that it lands heads up exactly 4 times? Express your answer as a common fraction.

136. Micah is at a donut shop and wants to choose his own dozen. There are ample glazed, chocolate and jelly donuts. If Micah wants at least two jelly donuts, how many different combinations of donuts can Micah use to create a box of a dozen donuts?

182. Skipper follows a randomly chosen path from A to C along the grid shown, always moving either up or to the right. If each path from A to C is equally likely to be chosen, what is the probability that his path goes through B? Express your answer as a common fraction.



187. An urn contains 5 red, 6 blue, 3 white and 8 yellow tokens. Shamus randomly selects two tokens, without replacement. What is the probability that he will draw two red tokens? Express your answer as a common fraction.

206. Two standard six-sided dice are rolled. What is the probability that their absolute difference is odd? Express your answer as a common fraction.

214. The function f has the property that $f(2x) = 8x^2 - 6x + 11$ for every x. What is the value of $f(4)$?

220. There are 30 balls, each a different color, and 30 boxes in the same 30 colors. One ball is placed in each box. How many ways can the balls be placed so that exactly 3 of the balls do not match the color of the box they are in?

222. Nine fair coins are flipped. What is the probability that the total number of coins that land heads up is a multiple of 3? Express your answer as a common fraction.

Problem #

224. Danny and Mila are playing tic-tac-toe. Their game board is shown. A fly lands randomly on one of the squares of their game board and then randomly moves to one of the squares that is vertically or horizontally adjacent to the square it landed on. What is the probability that the fly ends on a square marked with an X? Express your answer as a common fraction.

O	X	X
	O	

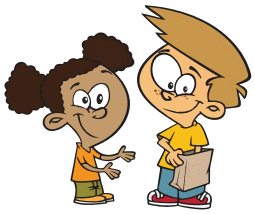
231. A two-digit integer is randomly selected. What is the probability that its digits are different? Express your answer as a common fraction.

233. Suppose 100 coins are flipped. If H is the number of ways that exactly 48 of them can land heads up, how many zeros are to the right of the rightmost nonzero digit of H?

236. A group of 5 horses is to be chosen from a stable of 20 horses to participate in the annual Derby Day Race. How many different groups of horses can be chosen from the stable, assuming that the order of the horses does not matter?

240. Three pairs of sisters stand in a line in a random order. What is the probability that everybody in the line is adjacent to her sister? Express your answer as a common fraction.

245. In a bag are five tokens, each marked with a different one of the numbers 1 through 5. Lorenzo randomly draws one token and places it back in the bag. Tamara then randomly draws a token from the bag. What is the expected value of the product of the numbers on the tokens that Lorenzo and Tamara draw?



250. Stacia has five pairs of gloves, each pair being a different color. She washed them all and now wants to match up the pairs. She randomly pairs each left glove with a right glove. If all five pairs do not match, she randomly pairs them all up again, and repeats until all five pairs do match. What is the expected number of times she will pair the gloves until all of them match?

Problem #

53. Hendrik rents a van for 4 days and drives it 180 miles. He is charged $40 per day and $0.10 per mile driven. What is the total cost of Hendrik's van rental?

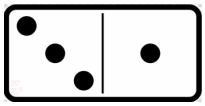
57. 

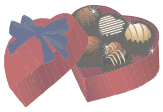
Mrs. Green wants to buy 60 stickers to give to her students. Stickers 'R' Us sells stickers at 3 for 45¢; World o' Stickers sells stickers at 5 for 65¢. If Mrs. Green purchases 60 stickers at the cheaper price, how much less will she pay than if she purchases 60 stickers at the more expensive price?

64. Chuck has decided to earn money by opening a hot chocolate stand. He spends $30 on supplies, and he plans on selling 40 cups of hot chocolate. How many dollars will he need to charge for each cup of hot chocolate to make a profit of $50?

78. 
Prya had a certain number of pencils at the beginning of school. She gave her brother half of the pencils. Then she gave Joy a third of the remaining pencils, after which she gave Ron two. If Prya was left with two pencils, how many pencils did she have at the beginning of school?

131. A set of double-six dominoes consists of 28 dominoes. Each domino has two ends, with 0 to 6 dots on each end and every possible combination occurring on exactly one domino. For example, there is exactly one of the domino shown, which has 3 dots on one side and one dot on the other. How many total dots are there on all 28 dominoes?



148. 
A specialty candy is packaged in a box holding only 1 candy, in a box holding 5 candies or in a box holding 16 candies. The company receives an order for 238 candies. What is the least number of boxes needed to exactly fill this order?

164. Jen finishes reading a book every 2 days, Ben every 5 days and Len every 7 days. If they all started a new book on 7/1 (July 1), the next date on which all three will start a new book is *M*/*D*, where *M* represents the number of the month and *D* represents the number of the day. What is the value of $M + D$?

177. Jessie and Henry are playing a game with standard six-sided dice. Each player rolls four dice. Henry rolls 5, 5, 2, 2, and Jessie rolls 4, 4, 4, 1. A player earns 2 points for each combination of dice that adds to 9. Any die can be used in more than one combination, but can only be used once in each combination. On this turn, Henry has two combinations that add to 9, (5, 2, 2) and (5, 2, 2), so he earns 4 points. How many points does Jessie earn?

201. Cassie sorted her beads into 100 piles. The first pile had 1 bead; the second pile had 3 beads; the third pile had 5 beads, and so on, with each pile having two more beads than the one before. Stacy sorted her beads into 100 piles, each containing an equal number of beads. If Stacy has the same total number of beads as Cassie, how many beads did Stacy put in each pile?

Problem #

42. Marion ran for 57 minutes at a pace of 10 minutes per mile. How many miles did she run? Express your answer as a decimal to the nearest tenth.

72. Winston rides his bike at a constant speed of 15 mi/h. Julia rides her bike at a constant speed of 18 mi/h. How many more seconds than Julia does it take Winston to ride 1 mile?

84. Kwesi and Kofi harvested enough honey to fill 11 quart-sized jars, 4 pint-sized jars and 7 half-cup jars. How many gallon-sized containers would it take to store the same amount of honey?

95. Driving at 50 mi/h, how many minutes does it take to drive 15 miles?

101.  A fir tree is $12\frac{1}{2}$ feet tall. How many inches tall is it?

111. Working together, Violet and Benny can pick 3 quarts of strawberries in 30 minutes. How many hours does it take them to pick 12 quarts of strawberries?

126. At the farmer's market, Yasmin sells one jar of za'atar every 8 minutes. Each jar sells for $8.80. Yasmin sells the first jar 8 minutes after the market opens at 7 a.m. What will Yasmin's total revenue be from selling za'atar when the market closes at 1 p.m.?

145. Michelle charges $45 per hour for tutoring. She is saving up to buy a $14,000 car. How many full hours must she tutor to earn enough money to purchase the car?

147. A certain burger chain sells, on average, 75 burgers every second. How many burgers are sold each hour? Express your answer in scientific notation to two significant figures.

152. At Adhira's grocery store, green cardamom pods cost $45.99 per pound. Given that there are 16 ounces in a pound, how much will 0.70 ounce of green cardamom pods cost?

154. At Bruno's shipping warehouse, any item weighing 5 pounds or less costs $3.50 to ship. For items over 5 pounds, the item costs $3.50 plus $1.25 for each additional pound. If Rachel paid $14.75 to ship a package, how many pounds did it weigh?

157. It takes 46 seconds to fill a 2-ounce measuring cup with ingredient A and 33 seconds to fill the same measuring cup with ingredient B. What is the absolute difference in the numbers of minutes it takes to fill the measuring cup with ingredients A and B? Express your answer as a decimal to the nearest hundredth.

160. Given that there are 2.54 cm in an inch and 5280 feet in a mile, how many centimeters are in a mile? Express your answer to the nearest thousand.

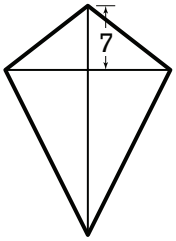
175. An entire bag of flour is enough to make 4 cakes. If Benji's Bakery has to fill an order for 81 cakes, what is the minimum number of bags of flour that must be opened?

Problem #

178. If Sandy can mow the yard in 1.5 hours and Chase can mow the same yard in 45 minutes, how many minutes will it take them to do the job together?

185. If 3 lemons cost $1.02, how much will 11 lemons cost?

191. Three people, each working at the same rate, can finish a building project in 8 days. How many days would it take 4 people working at the same rate to finish the project?

210. 

Marcus and Malia have a kite in the shape of a kite with diagonals of 25 cm and 18 cm. The shorter portion of the longer strut along a diagonal is 7 cm, as shown. They want to add a new strut, parallel to the shorter diagonal, that divides the kite into two equal areas. How long will that strut be?

238. Madison creates a batch of orange paint by mixing 1 pint of red paint with 1.5 quarts of yellow paint. How many more gallons of yellow paint does she need to add to the mixture to produce a 2-gallon batch of orange paint? Express your answer as a common fraction.

Sp Sequences, Series + Patterns

Problem #

113. Using consecutive numbers starting at 1, it takes 198 digits to number the pages in a book. How many pages are in the book?

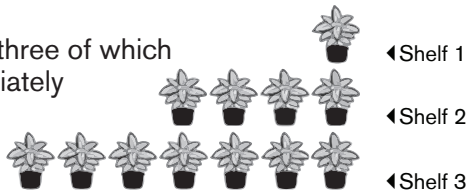
125. In a geometric sequence of positive numbers, if the 1015th term is 245 and the 3015th term is 45, what is the value of the 2015th term?

162. In an arithmetic sequence, the first four terms are 5, 11, 17 and 23. What is the 50th term in the sequence?

203. Assuming the pattern continues, what is the value of the infinite sum $3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32} + \cdots$?

212. Consider the sequence of numbers 1, 4, 13, 40, 121, . . ., where each number is one more than three times the previous number. The absolute difference between the 49th and 50th numbers in the sequence can be written as p^a, where p is a prime number and a is an integer. What is the value of $p + a$?

216. The plants at the garden center are placed on 15 shelves, the first three of which are as shown. Each shelf holds 3 more plants than the shelf immediately above it. In total, how many plants are on the 15 shelves?



221. In the product $\frac{2}{8} \times \frac{4}{10} \times \frac{6}{12} \times \frac{8}{14} \times \cdots \times \frac{94}{100}$, the numerator and denominator of each term are two greater than in the previous term. What is the value of the product? Express your answer as a common fraction.

Problem #

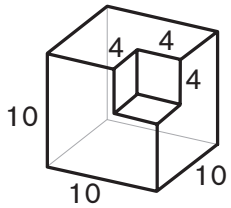
228. If the numbers 3, a, b and 375, in that order, form a geometric sequence, what is the value of $a + b$?

244. At the end of Day 1, Larry has \$40 and Moe has \$9. Every day, Larry spends \$1.50 while Moe saves \$1.50. What is the first day Moe will end with more money than Larry?

Sg Solid Geometry

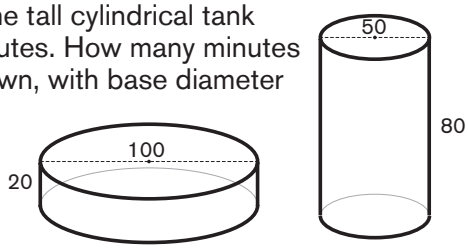
Problem #

47. A cube has side length 8 cm. A rectangular prism is formed by doubling the length of the cube, halving the width, and tripling the height. The volume of the rectangular prism is how many times that of the original cube?

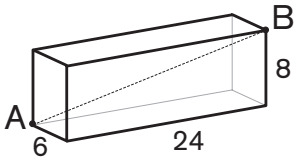
50. 

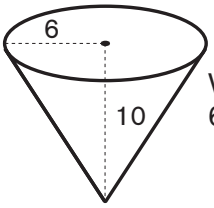
A cube of edge length 4 units is removed from one corner of a cube of edge length 10 units, as shown. What is the surface area of the resulting figure?

88. A water hose that discharges water at a constant rate can fill the tall cylindrical tank shown, with base diameter 50 cm and height 80 cm, in 12 minutes. How many minutes would it take the same hose to fill the short cylindrical tank shown, with base diameter 100 cm and height 20 cm?



144. What is the distance between point A and point B in the rectangular prism shown, with length 24 cm, width 6 cm and height 8 cm?



156. 

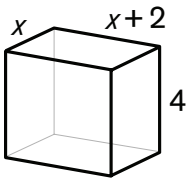
What is the volume of the cone shown here, with height 10 cm and base radius 6 cm? Express your answer in terms of π.

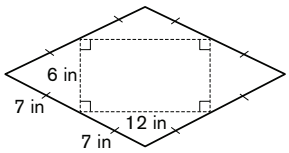
169. 

The formula for the volume V of a regular icosahedron of edge length a is $V = \frac{5(3+\sqrt{5})}{12}a^3$. What is the edge length of a regular icosahedron whose volume is 24.8 cm^3? Express your answer as a decimal to the nearest hundredth.

Problem #

196. The rectangular prism shown has length $(x + 2)$ units, width x units and height 4 units. If the prism has volume 80 units^3, what is the value of x? Express your answer in simplest radical form.



211. 

The net of a certain rectangular pyramid is composed of a 6-inch by 12-inch rectangle surrounded by two pairs of congruent isosceles triangles as shown. Each triangle has two sides of length 7 inches. What is the volume of the pyramid?

229. A certain cube has volume n ft^3 and surface area n in^2. If the integer n is written in the form $n = 2^a \cdot 3^b$ for integers a and b, what is the value of $a + b$?

243. Allan uses 19.11 square meters of sheet metal to construct a cylindrical silo with a bottom but no top. The barrel is 1.21 meters tall. What is the total volume of the silo? Express your answer as a decimal to the nearest hundredth.

Sd Statistics + Data

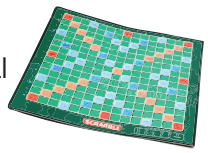
Problem #

1. Simone's average score for seven tests is 82 points. If she scores 90 points on her eighth test, what is the average of all eight test scores?

2. The average of Geoffrey's first four test scores is 72 points. If Geoffrey's fifth test score is 15 points more than the average of his first four test scores, what is the average of all five test scores?

3. The mean of four numbers is 18. If one of the four numbers is removed, the mean of the three remaining numbers is 17. What is the value of the number that was removed?

4. Abel, Bilal and Cara played a game of Scrabble. The average of the points scored by Abel and Bilal was 261 points, while the average number of points scored by Abel, Bilal and Cara was 269 points. How many points did Cara score?



5. Sage has 12 pennies and 8 nickels. The average mass of Sage's coins is 3.5 grams. If the average mass of Sage's pennies is 2.5 grams, what is the average mass of Sage's nickels?

6. 

Ruby mailed three packages to a friend. The mean weight of the packages was 85 ounces. When Ruby sent a fourth package, the mean weight increased by 2 ounces. How many ounces did the final package weigh?

7. Six cocker spaniels have a total weight of 192 pounds. Five golden retrievers have an average weight of 71 pounds. What is the average weight of all 11 dogs? Express your answer as a decimal to the nearest tenth.

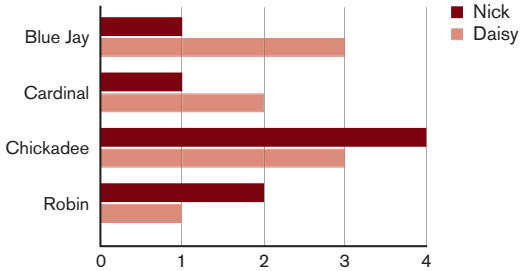
Problem #

8. Gil read 21 pages of a book on Monday, 34 pages on Tuesday, 17 pages on Wednesday, and 12 pages on Thursday. On Friday, Gil read 5 pages more than the mean number of pages read on the first four days. How many pages did Gil read in all?

9. Sam scored 50 points on the first of six Spanish exams in the semester. If Sam's goal is to have an average exam score of 90 points at the end of the semester, how many points will Sam need to score, on average, on the remaining five exams?

10. After taking three of the four exams in history class, Srinivasa has an average exam score of 66 points. If the fourth exam counts twice as much as the other exams, what is the fewest points Srinivasa can score on the fourth exam to pass the course with an overall exam average of at least 70 points?

32. Daisy and Nick went bird-watching. The chart shows the numbers of birds they saw of each species. How many more birds did Daisy see than Nick?



56. The digits 3, 4 and 8 can be used to make six different three-digit numbers in which each digit is used exactly once. What is the average of these six numbers?

69. A data set is displayed in the stem-and-leaf plot shown. If two additional data points, 24 and 29, are added to the data set, what will be the absolute difference between the new median and the original median?

1	0 0 1 3 9
2	1 1 1 1 5
3	3 5 7 7 8

Key: 2 | 1 = 21

74. If the mean of the data set $\{1, x - 1, 2x + 2, 3x - 1, 4x + 4\}$ is 7, what is the value of x?

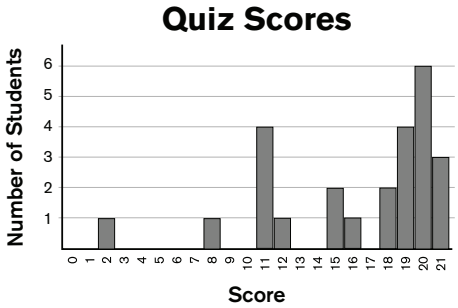
146. The mean of 12 scores on a math test was 86.9. Later, 3 more people take the test, and the average of their 3 scores is 90.0. What is the average of all 15 test scores? Express your answer as a decimal to the nearest tenth.

153. Novellor Technologies has 96 employees. The number of people filling each position and each position's mean annual salary are shown in the table. Given that the mean annual salary at Novellor is $102,000, what is the annual salary of Novellor's CEO? Express your answer to the nearest dollar.

Job	Number of Employees	Mean Annual Salary
Factory Worker	80	$42,000
Plant Manager	10	$64,500
Sales Rep	5	$51,000
CEO	1	

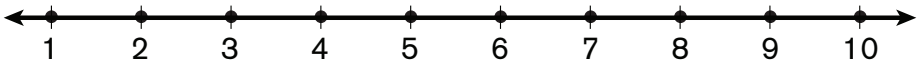
165. What is the absolute difference between the mean and the median of the numbers in the set {9, 8, 3, 4, 0, 20}? Express your answer as a decimal to the nearest tenth.

174. The bar graph shows the quiz scores received by the students in Ms. Novak's Civics class. What was the mean quiz score? Express your answer as a decimal to the nearest hundredth.



Problem #

237. The numbers 1 through 10 are marked with points on the number line as shown. Two distinct points are chosen at random from these points. What is the expected value of the distance between the two points? Express your answer as a decimal to the nearest tenth.



247. What is the absolute difference between the arithmetic mean and the geometric mean of 10 and 12? Express your answer as a decimal to the nearest hundredth.

COMPETITION COACH TOOLKIT

This is a collection of lists, formulas and terms that Mathletes frequently use to solve problems like those found in this handbook. There are many others we could have included, but we hope you find this collection useful.

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50
$\frac{1}{3}$	$0.\overline{3}$	$33.\overline{3}$
$\frac{1}{4}$	0.25	25
$\frac{1}{5}$	0.2	20
$\frac{1}{6}$	$0.1\overline{6}$	$16.\overline{6}$
$\frac{1}{8}$	0.125	12.5
$\frac{1}{9}$	$0.\overline{1}$	$11.\overline{1}$
$\frac{1}{10}$	0.1	10
$\frac{1}{11}$	$0.\overline{09}$	$9.\overline{09}$
$\frac{1}{12}$	$0.08\overline{3}$	$8.\overline{3}$

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

Common Arithmetic Series

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2$$

$$2 + 4 + 6 + 8 + \cdots + 2n = n^2 + n$$

Combinations & Permutations

$${}_nC_r = \frac{n!}{r!(n-r)!} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

Geometric Mean

$$\frac{a}{x} = \frac{x}{b} \quad \text{and} \quad x = \sqrt{ab}$$

Prime Numbers

2	43
3	47
5	53
7	59
11	61
13	67
17	71
19	73
23	79
29	83
31	89
37	97
41	

Divisibility Rules

2: units digit is 0, 2, 4, 6 or 8

3: sum of digits is divisible by 3

4: two-digit number formed by tens and units digits is divisible by 4

5: units digit is 0 or 5

6: number is divisible by both 2 and 3

8: three-digit number formed by hundreds, tens and units digits is divisible by 8

9: sum of digits is divisible by 9

10: units digit is 0

Equation of a Line

Standard Form

$$Ax + By = C$$

Slope-Intercept Form

$$y = mx + b$$

m = slope $\qquad$ b = y-intercept

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

m = slope $\qquad$ (x_1, y_1) = point on the line

Distance Traveled

Distance = Rate × Time

Quadratic Formula

For $ax^2 + bx + c = 0$, where $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagorean Triples

(3, 4, 5)	(5, 12, 13)	(7, 24, 25)
(8, 15, 17)	(9, 40, 41)	(12, 35, 37)

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Sum and Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Circles

Circumference	Area
$2 \times \pi \times r = \pi \times d$	$\pi \times r^2$

For radius r

Arc Length	Sector Area
$\frac{x}{360} \times 2 \times \pi \times r$	$\frac{x}{360} \times \pi \times r^2$

For central angle of x degrees

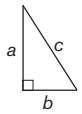
Given A(x_1, y_1) and B(x_2, y_2)

Distance from A to B = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of $\overline{AB}$ = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

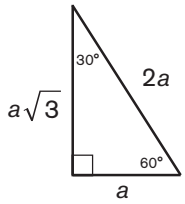
Slope of $\overline{AB}$ = $\frac{y_2 - y_1}{x_2 - x_1}$

Pythagorean Theorem

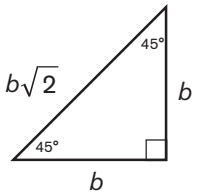


$a^2 + b^2 = c^2$

Special Right Triangles



30-60-90 Right Triangle



45-45-90 Right Triangle

Area of Polygons

Square	side length s	s^2
Rectangle	length l, width w	$l \times w$
Parallelogram	base b, height h	$b \times h$
Trapezoid	bases b_1, b_2, height h	$\frac{1}{2}(b_1 + b_2) \times h$
Rhombus	diagonals d_1, d_2	$\frac{1}{2} \times d_1 \times d_2$
Triangle	base b, height h	$\frac{1}{2} \times b \times h$
Triangle *Heron's formula*	semiperimeter s, side lengths a, b, c	$\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral Triangle	side length s	$\frac{s^2\sqrt{3}}{4}$

Polygon Angles (n sides)

Sum of the interior angle measures:

$$180 \times (n - 2)$$

Central angle measure of a regular polygon:

$$\frac{360}{n}$$

Interior angle measure of a regular polygon:

$$\frac{180 \times (n - 2)}{n} \quad \text{or} \quad 180 - \frac{360}{n}$$

Solid	Dimensions	Surface Area	Volume
Cube	side length s	$6 \times s^2$	s^3
Rectangular Prism	length l, width w, height h	$2 \times (l \times w + w \times h + l \times h)$	$l \times w \times h$
Cylinder	circular base radius r, height h	$2 \times \pi \times r \times h + 2 \times \pi \times r^2$	$\pi \times r^2 \times h$
Cone	circular base radius r, height h	$\pi \times r^2 + \pi \times r \times \sqrt{r^2 + h^2}$	$\frac{1}{3} \times \pi \times r^2 \times h$
Sphere	radius r	$4 \times \pi \times r^2$	$\frac{4}{3} \times \pi \times r^3$
Pyramid	base area B, height h		$\frac{1}{3} \times B \times h$

Vocabulary & Terms

The following list is representative of terminology used in the problems but **should not** be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference
absolute value
acute angle
additive inverse *(opposite)*
adjacent angles
apex
arithmetic mean
arithmetic sequence
base ten
binary
bisect
box-and-whisker plot
center
chord
circumscribe
coefficient
collinear
common divisor
common factor
common fraction
complementary angles
congruent
convex
coordinate plane/system
coplanar
counting numbers
counting principle
diagonal of a polygon
diagonal of a polyhedron
digit sum
dilation
direct variation
divisor
domain of a function
edge
equiangular
equidistant
expected value
exponent
exterior angle of a polygon
factor
finite
frequency distribution
frustum
function
GCF (GCD)
geometric sequence
hemisphere
image(s) of a point(s) *(under a transformation)*
improper fraction
infinite series
inscribe
integer
interior angle of a polygon
intersection
inverse variation
irrational number
isosceles
lateral edge
lateral surface area
lattice point(s)
LCM
median of a data set
median of a triangle
mixed number
mode(s) of a data set
multiplicative inverse *(reciprocal)*
natural number
obtuse angle
ordered pair
origin
palindrome
parallel
Pascal's Triangle
percent increase/decrease
perpendicular
planar
polyhedron
polynomial
prime factorization
principal square root
proper divisor
proper factor
proper fraction
quadrant
quadrilateral
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal *(multiplicative inverse)*
reflection
regular polygon
relatively prime
revolution
right angle
right polyhedron
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
semiperimeter
sequence
set
significant digits
similar figures
slope
space diagonal
square root
stem-and-leaf plot
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
transformation
translation
triangle inequality
triangular numbers
trisect
twin primes
union
unit fraction
variable
whole number
y-intercept

FORMS OF ANSWERS

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not have a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle of diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc* or *a.bc* (dollars), where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c are both zero, in which case they both may be omitted. Answers in the form ($) *a.bc* or *a.bc* (dollars) should be rounded to the nearest cent, unless otherwise specified. Examples:

Acceptable Forms: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:

Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3

Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

PROBLEM INDEX

It is very difficult to categorize many of the problems in this handbook. MATHCOUNTS problems often straddle multiple categories and cover several concepts, but in this index, we have placed each problem in exactly one category and mapped it to exactly one Common Core State Standard (CCSS). **In this index, code 9 (3) 7.SP.3 would refer to problem #9 with difficulty rating 3 mapped to CCSS 7.SP.3.** The difficulty rating and CCSS mapping are explained below.

DIFFICULTY RATING: Our scale is 1-7, with 7 being most difficult. These general ratings are only approximations:

- **1, 2 or 3**: Appropriate for students just starting the middle school curriculum; 1 concept; 1- or 2-step solution.
- **4 or 5**: Knowledge of some middle school topics necessary; 1-2 concepts; multi-step solution.
- **6 or 7**: Knowledge of advanced middle school topics and/or problem-solving strategies necessary; multiple and/or advanced concepts; multi-step solution.

COMMON CORE: We align our problems to the NCTM Standards for Grades 6-8, however we also have mapped these problems to CCSS because 41 states, D.C., 4 territories and the Dept. of Defense Education Activity (DoDEA) have voluntarily adopted it. Our CCSS codes contain (in this order):

1. **Grade level** in the K-8 Standards for Mathematical Content (SMC). Courses that are in the high school SMC instead have the first letter of the course name.
2. **Domain** within the grade level or course and then the **individual standard**.

Here are 2 examples:

- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts are not specifically mentioned in CCSS. For problems using these concepts, we use the code of a related standard, when possible. Some of our problems are based on concepts outside the scope of CCSS or are based on concepts in the K-5 SMC but are more difficult than a grade K-5 problem. When appropriate, we coded these problems SMP for the CCSS Standards for Mathematical Practice.

Sd

STATISTICS

1	(2)	6.SP.5
2	(2)	6.SP.5
3	(3)	6.SP.5
4	(3)	6.SP.5
5	(3)	6.SP.5
6	(3)	6.SP.5
7	(4)	6.SP.5
8	(3)	6.SP.5
9	(3)	6.SP.5
10	(3)	6.SP.5
32	(1)	6.SP.4
56	(5)	6.SP.5
69	(4)	6.SP.2
74	(3)	6.SP.5
146	(3)	6.SP.5
153	(3)	8.SP.4
165	(4)	6.SP.5
174	(4)	6.SP.4
237	(6)	S-MD.2
247	(3)	6.SP.5

Pr

PROPORTIONAL REASONING

42	(2)	6.RP.3
72	(4)	6.RP.3
84	(3)	7.RP.1
95	(2)	6.RP.3
101	(1)	5.MD.1
111	(2)	6.RP.3
126	(3)	6.RP.3
145	(2)	4.OA.3
147	(3)	6.RP.3
152	(2)	6.RP.2
154	(3)	SMP
157	(3)	6.RP.1
160	(3)	5.MD.1
175	(2)	4.OA.3
178	(4)	6.RP.3
185	(2)	6.RP.2
191	(4)	6.RP.3
210	(6)	6.G.1
238	(4)	6.RP.3

Ps

PROBLEM SOLVING (MISCELLANEOUS)

53	(3)	SMP
57	(3)	SMP
64	(2)	SMP
78	(3)	SMP
131	(3)	SMP
148	(3)	SMP
164	(4)	SMP
177	(3)	SMP
201	(5)	SMP

Sp

SEQUENCES, SERIES & PATTERNS

113	(3)	SMP
125	(6)	F-BF.2
162	(4)	F-BF.2
203	(5)	F-BF.2
212	(5)	F-BF.2
216	(4)	F-IF.3
221	(4)	7.NS.2
228	(5)	F-BF.2
244	(3)	SMP

Lo

LOGIC

61	(3)	SMP
70	(4)	SMP
100	(4)	SMP
103	(4)	SMP
142	(2)	SMP
192	(4)	SMP
232	(3)	SMP
234	(4)	SMP

Gm

GENERAL MATH

33	(2)	4.OA.3
34	(1)	4.OA.3
39	(2)	4.NBT.5
43	(2)	SMP
49	(2)	4.OA.5
79	(2)	7.NS.2
141	(1)	4.OA.3

Cg

COORDINATE GEOMETRY

20	(6)	7.G.6
82	(4)	G-GPE.7
92	(4)	G-GPE.7
119	(6)	F-IF.7
163	(5)	G-GPE.5
198	(4)	8.EE.8
218	(5)	8.G.8

Pc

PROBABILITY, COUNTING & COMBINATORICS

19	(4)	7.G.6
21	(2)	S-CP.9
22	(3)	S-CP.9
23	(3)	S-CP.9
24	(3)	S-CP.9
25	(5)	S-CP.9
26	(4)	S-CP.9
27	(5)	S-CP.9
28	(6)	S-CP.9
29	(5)	S-CP.9
30	(5)	S-CP.9
38	(4)	7.SP.7
55	(4)	7.SP.7
76	(5)	S-CP.9
87	(4)	7.SP.8
94	(5)	7.SP.8
98	(4)	7.SP.7
105	(3)	SMP
110	(4)	S-CP.2
115	(3)	S-CP.9
117	(4)	S-CP.8
122	(2)	7.SP.7
127	(3)	S-CP.9
129	(5)	S-CP.9
135	(6)	S-CP.9
136	(5)	S-CP.9
182	(5)	S-CP.9
187	(5)	S-CP.8
206	(4)	7.SP.8
214	(5)	F-IF.2
220	(6)	S-CP.9
222	(6)	S-CP.7
224	(6)	S-CP.7
231	(3)	S-CP.6
233	(6)	S-CP.9
236	(5)	S-CP.9
240	(4)	S-CP.9
245	(6)	S-MD.2
250	(7)	S-MD.2

Me

MEASUREMENT

16	(3)	7.G.4
31	(2)	G-GPE.7
41	(1)	SMP
46	(4)	8.G.7
51	(2)	5.MD.1
60	(5)	G-GPE.7
77	(3)	6.G.1
91	(3)	7.G.4
104	(3)	7.G.6
116	(4)	G-C.5
159	(5)	G-SRT.6
170	(2)	7.G.6
176	(3)	7.NS.2
186	(4)	7.G.6
193	(1)	5.MD.1
197	(5)	7.G.6
200	(5)	G-GMD.3
242	(5)	7.G.6

Al

ALGEBRAIC EXPRESSIONS & EQUATIONS

35	(2)	6.EE.6
36	(4)	6.EE.3
37	(3)	6.EE.6
48	(4)	8.EE.8
54	(4)	6.EE.6
58	(2)	6.EE.6
65	(3)	6.EE.6
71	(2)	6.EE.3
73	(4)	8.EE.8
75	(4)	6.EE.7
90	(5)	8.EE.1
99	(4)	7.EE.4
102	(4)	A-REI.11
106	(3)	6.EE.6
108	(4)	6.RP.3
114	(5)	6.EE.6
132	(5)	A-REI.11
134	(5)	6.EE.6
149	(4)	6.EE.7
150	(2)	6.EE.1
155	(5)	6.RP.3
180	(4)	6.EE.2
184	(3)	6.EE.6
219	(5)	A-REI.4
239	(6)	SMP
241	(4)	SMP
249	(6)	8.EE.2

Sg

SOLID GEOMETRY

47	(4)	7.G.6
50	(4)	7.G.6
88	(5)	G-GMD.3
144	(4)	8.G.8
156	(4)	G-GMD.3
169	(4)	6.EE.2
196	(5)	G-GMD.3
211	(6)	6.G.4
229	(5)	6.EE.7
243	(5)	8.G.9

Nt

NUMBER THEORY

40	(3)	4.OA.4
45	(2)	4.OA.4
52	(2)	6.NS.3
59	(4)	SMP
62	(3)	8.EE.1
63	(3)	6.NS.4
66	(4)	6.EE.3
81	(2)	6.EE.9
85	(4)	8.EE.1
86	(3)	6.NS.2
97	(3)	6.EE.7
107	(3)	8.EE.1
109	(4)	SMP
112	(5)	N-RN.2
118	(4)	SMP
120	(4)	SMP
124	(4)	SMP
128	(5)	6.NS.4
130	(5)	SMP
137	(4)	SMP
171	(5)	SMP
172	(3)	A-CED.1
173	(5)	SMP
183	(5)	SMP
199	(4)	SMP
202	(3)	SMP
204	(3)	6.EE.2
207	(5)	6.NS.4
208	(4)	SMP
209	(4)	SMP
215	(5)	6.NS.4
217	(3)	SMP
225	(5)	SMP
226	(4)	8.NS.1
246	(4)	6.NS.4

Pg

PLANE GEOMETRY

11	(3)	7.G.4
12	(2)	7.G.4
13	(2)	8.G.3
14	(4)	7.G.4
15	(4)	7.G.6
17	(6)	7.G.6
18	(5)	7.G.6
68	(4)	G-SRT.6
80	(3)	8.G.5
89	(4)	G-CO.10
93	(4)	G-CO.10
121	(5)	G-C.2
123	(5)	8.G.5
133	(5)	SMP
138	(5)	7.G.6
139	(5)	7.G.6
140	(6)	SMP
179	(5)	7.G.6
188	(4)	7.G.6
195	(6)	G-C.2
205	(5)	7.G.5
213	(6)	G-C.2
223	(4)	SMP
227	(5)	G-SRT.6
230	(5)	G-C.5
235	(6)	G-C.5

Pf

PERCENTS & FRACTIONS

44	(3)	6.NS.6
67	(3)	6.RP.3
83	(2)	7.RP.3
96	(2)	7.NS.3
143	(2)	6.RP.3
151	(2)	6.RP.3
158	(2)	6.RP.3
161	(2)	6.RP.3
166	(3)	7.RP.3
167	(4)	6.RP.3
168	(2)	7.RP.3
181	(4)	6.RP.3
189	(3)	6.RP.3
190	(3)	SMP
194	(2)	6.RP.3
248	(4)	6.RP.3

ANSWERS

Note that these answers are arranged by problem number, *not* by topic, for ease of searching. Step-by-step solutions to these problems are available to registered Competition Series Coaches online or in the print version of the School Handbook available for purchase at mathcountsstore.com.

1.	83	(2)	51.	6	(2)	101.	150	(1)	151.	40	(2)	201.	100	(5)
2.	75	(2)	52.	95	(2)	102.	8/5	(4)	152.	2.01	(2)	202.	9600	(3)
3.	21	(3)	53.	178 or	(3)	103.	130	(4)	153.	5,532,000	(3)	203.	6	(5)
4.	285	(3)		178.00		104.	42	(3)		or 5,532,000.00		204.	14	(3)
5.	5	(3)	54.	11/6	(4)	105.	15	(3)	154.	14	(3)	205.	6	(5)
6.	93	(3)	55.	1/2	(4)	106.	2	(3)	155.	4.6	(5)	206.	1/2	(4)
7.	49.7	(4)	56.	555	(5)	107.	5	(3)	156.	120π	(4)	207.	29	(5)
8.	110	(3)	57.	1.20	(3)	108.	75	(4)	157.	0.22 or .22	(3)	208.	918	(4)
9.	98	(3)	58.	22	(2)	109.	13	(4)	158.	10	(2)	209.	13	(4)
10.	76	(3)	59.	72	(4)	110.	$2\frac{2}{3}$	(4)	159.	$4000\sqrt{2}$	(5)	210.	15	(6)
			60.	4.5	(5)				160.	161,000	(3)			
11.	34π	(3)	61.	4	(3)	111.	2	(2)	161.	20	(2)	211.	48	(6)
12.	13	(2)	62.	337/144	(3)	112.	1/2	(5)	162.	299	(4)	212.	52	(5)
13.	16	(2)	63.	17	(3)	113.	102	(3)	163.	9	(5)	213.	2/3	(6)
14.	0.6 or .6	(4)	64.	2	(2)	114.	1/4	(5)	164.	18	(4)	214.	31	(5)
15.	$25\pi/4$	(4)	65.	7/30	(3)	115.	1/6	(3)	165.	1.3	(4)	215.	48	(5)
16.	40	(3)	66.	70	(4)	116.	20π	(4)	166.	44.25	(3)	216.	330	(4)
17.	9	(6)	67.	1000	(3)	117.	7/31	(4)	167.	12	(4)	217.	1:24	(3)
18.	$8\sqrt{2}$	(5)	68.	$10\sqrt{3}$	(4)	118.	24	(4)	168.	11	(2)	218.	12	(5)
19.	1/16	(4)	69.	0	(4)	119.	$4\sqrt{37}$	(6)	169.	2.25	(4)	219.	−4	(5)
20.	82.5	(6)	70.	32	(4)	120.	7	(4)	170.	12.5	(2)	220.	8120	(6)
21.	220	(2)	71.	3	(2)	121.	48	(5)	171.	68	(5)	221.	1/19,600	(4)
22.	36	(3)	72.	40	(4)	122.	9/35	(2)	172.	16	(3)	222.	85/256	(6)
23.	286	(3)	73.	0	(4)	123.	104	(5)	173.	166,833	(5)	223.	9	(4)
24.	816	(3)	74.	3	(3)	124.	957	(4)	174.	16.28	(4)	224.	23/108	(6)
25.	70	(5)	75.	42	(4)	125.	105	(6)	175.	21	(2)	225.	900	(5)
26.	20	(4)	76.	90	(5)	126.	396 or	(3)	176.	29,700	(3)	226.	151	(4)
27.	16	(5)	77.	6.25	(3)		396.00		177.	6	(3)	227.	45	(5)
28.	200	(6)	78.	12	(3)	127.	13	(3)	178.	30	(4)	228.	90	(5)
29.	161	(5)	79.	1	(2)	128.	12	(5)	179.	4/3	(5)	229.	24	(5)
30.	56	(5)	80.	30	(3)	129.	18	(5)	180.	3.6	(4)	230.	45	(5)
						130.	400	(5)						
31.	72	(2)	81.	366	(2)	131.	168	(3)	181.	40	(4)	231.	9/10	(3)
32.	1*	(1)	82.	2	(4)	132.	3	(5)	182.	9/17	(5)	232.	90	(3)
33.	870	(2)	83.	4.3	(2)	133.	$35\frac{1}{2}$	(5)	183.	7:48	(5)	233.	2	(6)
34.	8	(1)	84.	4	(3)	134.	276	(5)	184.	−2	(3)	234.	18	(4)
35.	9	(2)	85.	2	(4)	135.	35/128	(6)	185.	3.74	(2)	235.	34.4	(6)
36.	−2	(4)	86.	11	(3)	136.	66	(5)	186.	29.54	(4)	236.	15,504	(5)
37.	65	(3)	87.	5/9	(4)	137.	113	(4)	187.	10/231	(5)	237.	3.7	(6)
38.	1/8	(4)	88.	12	(5)	138.	48	(5)	188.	18	(4)	238.	9/8	(4)
39.	350	(2)	89.	7	(4)	139.	6	(5)	189.	1*	(3)	239.	157	(6)
40.	42	(3)	90.	9	(5)	140.	$5\sqrt{3}/2$	(6)	190.	8	(3)	240.	1/15	(4)
41.	$5\frac{1}{4}$	(1)	91.	$4\pi^3$	(3)	141.	1.25	(1)	191.	6	(4)	241.	1	(4)
42.	5.7	(2)	92.	9	(4)	142.	120	(2)	192.	14	(4)	242.	15.2	(5)
43.	3.03	(2)	93.	23	(4)	143.	25	(2)	193.	9.21	(1)	243.	8.98	(5)
44.	7/36	(3)	94.	1/512	(5)	144.	26	(4)	194.	20	(2)	244.	12	(3)
45.	17	(2)	95.	18	(2)	145.	312	(2)	195.	6.9	(6)	245.	9	(6)
46.	28	(4)	96.	1.6	(2)	146.	87.5	(3)	196.	$-1 + \sqrt{21}$	(5)	246.	6	(4)
47.	3	(4)	97.	1	(3)	147.	2.7×10^5	(3)		or $\sqrt{21} - 1$		247.	0.05 or .05	(3)
48.	6	(4)	98.	12/17	(4)	148.	19	(3)	197.	3174	(5)	248.	3.3	(4)
49.	15,780	(2)	99.	2/9	(4)	149.	2	(4)	198.	5	(4)	249.	480 or	(6)
50.	600	(4)	100.	6	(4)	150.	150	(2)	199.	$2\frac{1}{2}$	(4)		480.00	
									200.	7.41×10^9	(5)	250.	120	(7)

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

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