MATHCOUNTS®

2025 Chapter Competition Solutions

Are you wondering how we could have possibly thought that a Mathlete® would be able to answer a particular Sprint Round problem without a calculator?

Are you wondering how we could have possibly thought that a Mathlete would be able to answer a particular Target Round problem in less 3 minutes?

Are you wondering how we could have possibly thought that a particular Team Round problem would be solved by a team of only four Mathletes?

The following pages provide detailed solutions to the Sprint, Target and Team Rounds of the 2025 MATHCOUNTS Chapter Competition. These solutions show creative and concise ways of solving the problems from the competition.

There are certainly numerous other solutions that also lead to the correct answer, some even more creative and more concise!

We encourage you to find a variety of approaches to solving these fun and challenging MATHCOUNTS problems.

Special thanks to solutions author
Howard Ludwig
for graciously and voluntarily sharing his solutions
with the MATHCOUNTS community.

Sprint 1
$2 + (2 \times 2) + (2 \times 2 \times 2) = 2 + 4 + 8 = \mathbf{14}$.

Sprint 2
There are 3 vowels (A, O, U), each worth 8, and 7 consonants (M, T, H, C, N, T, S), each worth 5. As a cross-check (it being easy to overlook a letter), let's count all the letters in MATHCOUNTS and verify it is $3 + 7 = 10$—yes, keep going. The total value is, therefore, $3 \times 8 + 7 \times 5 = 24 + 35 = \mathbf{59}$.

Sprint 3
$12x = 36$, so $x = 36/12 = \mathbf{3}$.

Sprint 4
Of the five values (21, 15, 12, 37, 48), only two (12, 48) are multiples of 6, so the probability is **2/5**.

Sprint 5
Let A, B. and E be the current age of Al, Ben, and Ed, respectively. $E - B = (A + 2) - (A - 5) = (A - A) + (2 + 5) = \mathbf{7}$.

Sprint 6
$\frac{12-♥}{3} = 1$. Multiply both sides by 3: $12 - ♥ = 3$. Add $♥ - 3$ to both sides: $\mathbf{9} = ♥$.

Sprint 7
$A = (2 \times 10 \text{ cm})(2 \times 15 \text{ cm}) = 20 \times 30 \text{ cm}^2 = \mathbf{600} \text{ cm}^2$.

Sprint 8
Because only positive values are involved, the least product is the product of the two least elements, 1 and 2, and the greatest product is the product of the two greatest elements, 3 and 4. Therefore, the desired sum is $1 \times 2 + 3 \times 4 = \mathbf{14}$.

Sprint 9
The figure suggests a linear relationship. When x shifts by 6 from 0 to 6, y shifts 4 from -4 to 0. Therefore, when x shifts another 6 to 12, y shifts another 4 from 0 to **4**.

Sprint 10
A sphere inscribed in a cube is tangent to each face of the cube. Any line segment connecting the points of tangency on opposite faces of the cube constitutes a diameter of the sphere. Therefore, both the diameter of the sphere and edge-length of the cube are the same, 6 cm. The radius of the circle is half that diameter, thus 3 cm. Therefore, the volume enclosed by the sphere is given by $V_{\text{sphere}} = \frac{4}{3}\pi(3 \text{ cm})^3 = \frac{4\pi \times 27}{3} \text{ cm}^3 = \mathbf{36\pi} \text{ cm}^3$.

Sprint 11
$\frac{15+7+12+5+x}{5} = 10$, so $x = 5 \times 10 - 15 - 7 - 12 - 5 = \mathbf{11}$.

Sprint 12
The volume of a rectangular prism is the square root of the product of the surface areas of three mutually adjacent faces (all three faces intersecting at one vertex). For any such set of three faces, one face has area 35 in^2, another has area 45 in^2, and yet another has area 63 in^2. The numeric parts have respective prime factorizations of $5 \times 7,\ 3^2 \times 5,\ 3^2 \times 7$. Therefore, we need $\sqrt{3^4 \times 5^2 \times 7^2 \,(\text{in}^2)^3} = 3^2 \times 5 \times 7 \text{ in}^3 = 9 \times 5 \times 7 \text{ in}^3 = 45 \times 7 \text{ in}^3 = \mathbf{315} \text{ in}^3$.

Copyright MATHCOUNTS, Inc. 2025. All rights reserved.

Sprint 13
The only way to use quarters in 1 quarter is with 1 nickel. Without quarters, there must be 0, 1, 2, or 3 dimes, with the only choice for nickels in each case being 6 minus twice the number of dimes. The total number of coin distributions is, therefore, **5**.

Sprint 14
Use the complement technique—shaded area is total area minus unshaded areas. The total area is $4 \text{ cm} \times 5 \text{ cm} = 20 \text{ cm}^2$. The upper left unshaded area is $3 \text{ cm} \times 3 \text{ cm}/2 = 4.5 \text{ cm}^2$. The lower left unshaded area is $1 \text{ cm} \times 5 \text{ cm}/2 = 2.5 \text{ cm}^2$. The upper right unshaded area is $4 \text{ cm} \times 2 \text{ cm}/2 = 4 \text{ cm}^2$. Therefore, the shaded area is $(20 - 4.5 - 2.5 - 4) \text{ cm}^2 = \mathbf{9} \text{ cm}^2$.

Sprint 15
The total count of marbles is $12 + 7 + 11 + 3 = 33$. The first selection is to be pink, which is 3 desire choices out of 33 total choices, so $\frac{3}{33} = \frac{1}{11}$. Now there are 32 marbles left to choose from, of which 12 are the desired purple, so $\frac{12}{32} = \frac{3}{8}$. These two events are independent, so the overall probability is the product of these two individual probabilities: $\frac{1}{11} \times \frac{3}{8} = \mathbf{\frac{3}{88}}$.

Sprint 16
The tank volume is $10 \text{ ft} \times 3 \text{ ft} \times 2 \text{ ft} = 60 \text{ ft}^3$. At 11:00 the water volume is $70\% = 0.7$ of the tank volume, thus $0.7 \times 60 \text{ ft}^3 = 42 \text{ ft}^3$. After 1 h, it is down to $0.4 \times 60 \text{ ft}^3 = 24 \text{ ft}^3$, thus dropping $42 \text{ ft}^3 - 24 \text{ ft}^3 = 18 \text{ ft}^3$ each hour. Therefore, after 1 h more, the water volume will drop another 18 ft^3 to $24 \text{ ft}^3 - 18 \text{ ft}^3 = \mathbf{6} \text{ ft}^3$.

Sprint 17
$2025 = 81 \times 25 = 3^4 \times 5^2$, so the requisite sum is $3 + 4 + 5 + 2 = \mathbf{14}$.

Sprint 18
Each path starts at 5. Then there are 4 choices: squares 2, 4, 6, or 8. For each of those, there are 2 adjacent corner squares that can be chosen. Then, no matter how you got to where you are, there is only 1 unused adjacent square. This yields a total of $4 \times 2 \times 1 = 8$ distinct paths. However, these 8 paths are paired based on covering the same four squares, thus having the same sum, and merely swapping which square is covered by move 1 versus move 3. This results in 4 distinct patterns for sum: the 2×2 squares in (1) the upper-left corner, (2) the upper-right corner, (3) the lower-left corner, (4) the lower-right corner. The sum of the respective sums is:
$(1 + 2 + 4 + 5) + (2 + 3 + 5 + 6) + (4 + 5 + 7 + 8) + (5 + 6 + 8 + 9) = 12 + 16 + 24 + 28 = \mathbf{80}$.

Sprint 19
The amount of increase in the average is the average increase in the ten values is $\frac{(1+2+3+4+5+6+7+8+9+10)}{10} = \frac{\cancel{10}\times 11/2}{\cancel{10}} = 5.5$, so the new average is $9.5 + 5.5 = \mathbf{15}$.

Sprint 20
The prime factorization of 64 is 2^6; that exponent 6 has 4 factors: 6, 3, 2, 1, so we have $(2^6)^1 = 64^1$; $(2^3)^2 = 8^2$; $(2^2)^3 = 4^3$; $(2^1)^6 = 2^6$. Desired sum is $(64 + 1) + (8 + 2) + (4 + 3) + (2 + 6) = \mathbf{90}$.

Copyright MATHCOUNTS, Inc. 2025. All rights reserved.

Sprint 21

The first three positive integers violate the triangle inequality theorem, thus do not form a triangle. With 2 and 3 available, the next length needs to be at least $2 + 3 = 5$ to violate the triangle inequality theorem, so pick 5 for bare minimum violation. Similarly, with 3 and 5 available, the last length needs to be at least $3 + 5 = 8$. We are generating the Fibonacci sequence starting with 1 and 2. The five needed values are 1; 2; 3; 5; 8. Two ways to add these: (1) There are only five values and they are small integers, so just add them. (2) The sum of a general contiguous finite subsequence of Fibonacci values is given by $F_m + F_{m+1} + \cdots + F_{n-1} + F_n = F_{n+2} - F_{m+1}$—if you have the Fibonacci sequence memorized to at least 21 or the problem involved more than five matches, this would be faster, but for this problem, we have $5 + 8 = 13; 8 + 13 = 21$, and the second one in our list is 2, so subtract $21 - 2 = 19$. Therefore, with all lengths being in centimeters, the desired sum is **19** cm.

Sprint 22

For $2^x = x + 3$, $2^x > 0$ for all real numbers x, so the equation requires $x + 3 > 0$. Dividing both sides of the equation by the nonzero value $x + 3$ yields $\frac{2^x}{x+3} = 1$, so

$$\frac{2^{2^x}}{x+3} = \frac{2^{(2^x)}}{x+3} = \frac{2^{(x+3)}}{x+3} = \frac{2^x \cdot 2^3}{x+3} = \frac{2^x}{x+3} \cdot 8 = 1 \times 8 = \mathbf{8}.$$

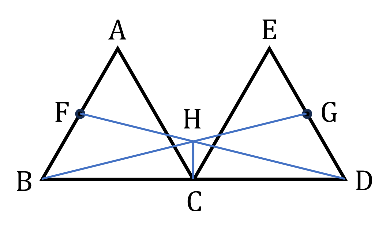
Sprint 23

Square both sides of $\sqrt{6 + 2\sqrt{5}} = a + \sqrt{b}$ yields $6 + 2\sqrt{5} = a^2 + b + 2a\sqrt{b}$, so $b = 5$ and $a = 1$. (With these values for a and b, $a + \sqrt{b} > 0$ and $\sqrt{\ }$ applied to a positive value always yields a positive value, so the signs of the two sides match and squaring both sides will not trigger any extraneous "solutions".) Therefore, $a + b = 1 + 5 = \mathbf{6}$.

Sprint 24

Because $n > 0$, $\frac{2}{3} \le \frac{m}{n} < 1$ is equivalent to $\frac{2}{3}n \le m < n$. Because m and n are integers, we can tighten the last pair of inequalities to $\left\lceil \frac{2}{3}n \right\rceil \le m \le n - 1$ (where $\lceil \square \rceil$ denotes the ceiling operator, the least integer greater than or equal to the one operand, which is enclosed between the two parts of the operator). Thus, the number of satisfactory values of m for any particular value of n is given by $n - \left\lceil \frac{2}{3}n \right\rceil$, which is positive only for $n \ge 3$. For n of the form of any of $3k, 3k + 1, 3k + 2$ for some integer k, there are k values of m that satisfy the constraints. For $3 \le n \le 10$, n ranges from $3(1)$ through $3(3) + 1$, yielding 3 with $k = 1$, 3 with $k = 2$, and 2 with $k = 3$. Therefore, there are in total $3 \times 1 + 3 \times 2 + 2 \times 3 = \mathbf{15}$ distinct (n, m) pairs satisfying the given constraints.

Sprint 25



Establish a coordinate system with the x-axis along $\overline{BD}$ and origin at C. All coordinates represent number of inches for x and y. Then A is at $(-7,\ 7\sqrt{3})$; B is at $(-14,\ 0)$; C is at $(0,\ 0)$; D is at $(+14,\ 0)$; E is at $(+7,\ 7\sqrt{3})$; F is at $(-10.5,\ 3.5\sqrt{3})$; G is at $(+10.5,\ 3.5\sqrt{3})$. The slope of $\overline{BG}$ is $\frac{3.5\sqrt{3}-0}{10.5-(-14)} = \frac{\sqrt{3}}{7}$. The horizontal distance traversed from B to H is $[0 - (-14)]$ in $= 14$ in; multiplying that by the slope $\frac{\sqrt{3}}{7}$ yields a y-coordinate at H of $0 + 2\sqrt{3} = 2\sqrt{3}$. Therefore, H is at $(0,\ 2\sqrt{3})$, which is a distance of $\mathbf{2\sqrt{3}}$ in above C.

Copyright MATHCOUNTS, Inc. 2025. All rights reserved.

Sprint 26

Let's define $a' = \frac{a-1}{2}$, $b' = \frac{b-1}{2}$, $c' = \frac{c-1}{2}$, so that we are dealing with consecutive nonnegative integers so as not having to compensate for skips, etc. By now requiring $a' + b' + c' = \frac{49-3}{2} = 23$ with $0 \le a' < b' < c'$, we have a parallel case with the same total counts. Now, $0 \le a' \le \left\lfloor \frac{23-3}{3} \right\rfloor = 6$. For $a' = 6$, (b', c') ranges from $(7, 10)$ to $(8, 9)$ [2 cases]. For $a' = 5$, (b', c') ranges from $(6, 12)$ to $(8, 10)$ [3 cases]. For $a' = 4$, (b', c') ranges from $(5, 14)$ to $(9, 10)$ [5 cases]. For $a' = 3$, (b', c') ranges from $(4, 16)$ to $(9, 11)$ [6 cases]. For $a' = 2$, (b', c') ranges from $(3, 18)$ to $(10, 11)$ [8 cases]. For $a' = 1$, (b', c') ranges from $(2, 20)$ to $(10, 12)$ [9 cases]. For $a' = 0$, (b', c') ranges from $(1, 22)$ to $(11, 12)$ [11 cases]. The total is $2 + 3 + 5 + 6 + 8 + 9 + 11 = \mathbf{44}$ valid ordered triples.

Sprint 27

The kth term of the series is $4!\ {}_{k+3}C_4$. The sum of the first m such terms is $4!\ {}_{m+4}C_5$. Thus, for $m = 25$, the sum is $4! \times \frac{29\times28\times27\times26\times25}{5!} = \frac{29\times28\times27\times26\times25}{5} = 29 \times 28 \times 27 \times 26 \times 5$. The 28 provides 2 factors of 2 and the 26 provides 1 factor of 2, for a total of 3 factors of 2. Therefore, $n = \mathbf{3}$.

Sprint 28

$76^2 - 75^2 = (76 + 75)(76 - 75) = 151 \times 1 = 151$. Similarly, $75^2 - 74^2 = 75 + 74 = 149$. Therefore, by linear interpolation, for $74^2 \le n \le 75^2$, $\sqrt{n+1}$ averages about 1/149 greater than $\sqrt{n}$; for $75^2 \le n \le 76^2$, $\sqrt{n+1}$ averages about 1/151 greater than $\sqrt{n}$. We have n right at 75^2, the boundary between these two intervals, so it is reasonable to expect that $\sqrt{75^2+1}$ is very close to 1/150 greater than $\sqrt{75^2} = 75$. Now, $\frac{1}{150} = \frac{1}{25} \times \frac{1}{6} = \frac{0.04}{6} = 0.00\overline{6}$, so $\sqrt{75^2+1} \approx 75.0067$ (true value is 75.006 666 3…]. The first nonzero digit to the right of the decimal mark is **6**.

Sprint 29

This technique is sometimes referred to as Simon's [Rubinstein-Salzedo] Favorite Factoring Trick. Let's adjust the satisfying equation by multiplying both sides by $(a + b)$ to get $7ab = 2025(a + b)$. Now expand the right side and shift both terms to the left side: $7ab - 2025a - 2025b = 0$. The equation is symmetric in a and b. We need to do some factoring somewhat similar to completing the square, but 7 is prime. To maintain the symmetry for factoring, let's multiply both sides of the equation by 7: $0 = 7^2ab - 7(2025a) - 7(2025b) = 7a(7b - 2025) - 2025(7b - 2025) - 2025^2$ $= (7a - 2025)(7b - 2025) - 2025^2$. Add 2025^2 to both sides of the equation: $(7a - 2025)(7b - 2025) = 2025^2 = 3^8 \times 5^4$. Now we need to decompose $3^8 \times 5^4$ into all possible pairs of factors whose product is $3^8 \times 5^4$ such that one factor corresponds to $(7a - 2025)$ and the other factor to $(7b - 2025)$. There are $(8 + 1)(4 + 1) = 45$ positive divisors of $3^8 \times 5^4$. One of them is 45, which pairs with itself. The remaining 44 divisors form 22 pairs if we ignore order of factors. If one factor in a pair matches up, then so does the other factor. Thus, we need consider only the 23 divisors less than or equal to $3^4 \times 5^2 = 2025$, and determine how many of them match, then double that count because for $a < b$ both (a, b) and (b, a) qualify as distinct solutions for this problem, except that if $(45, 45)$ works, then it is a special case yielding only one solution, not two. Each factor must match with one of $(7a - 2025)$ or $(7b - 2025)$, each satisfying $(7x - 2025) \equiv 5 \pmod 7$. Therefore, we will count those numeric divisors of $3^8 \times 5^4$ whose remainder upon dividing by 7 is 5. The divisors of $3^8 \times 5^4$ less than or equal to 2025 are, with those satisfying the modular congruence in italic: 1; 3; 9; 27; 81; *243*; 729; *5*; 15; 45; 135; 405; 1215; 25; *75*; 225; 675; 2025; 125; 375; *1125*; 625; 1875. There are 4 qualifying divisors, thus **8** suitable ordered pairs.

Copyright MATHCOUNTS, Inc. 2025. All rights reserved.

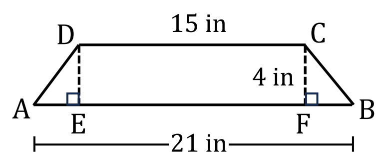
Sprint 30

Regard $\triangle ABC$ as the base as the tetrahedron and BD as its height, given that $\overline{BD}$ is perpendicular to the base. A tetrahedron is a pyramid composed of a triangular base and three lateral triangular sides, and its volume is 1/3 of the product of the area of the base times the height. Because $m\angle ABC = 90°$, the base is a right triangle and its area is $BA \cdot BC/2$. The given angular measures indicate that $\triangle ABD$ and $\triangle BCD$ are 30°-60°-90° triangles, making the longer leg $\sqrt{3}$ times as long the shorter leg. $m\angle DAB = 30°$ means BD is the shorter leg and BA is the longer leg, so $BA = \sqrt{3}BD$. $m\angle DCB = 60°$ means BC is the shorter leg and BD is the longer leg, so $BC = BD/\sqrt{3}$. Therefore, the volume of the tetrahedron is $\frac{1}{3}\left(\frac{1}{2}\ BA \cdot BC\right) \cdot BD = \frac{1}{6}\left[\left(\sqrt{3}\ BD\right) \cdot \left(\frac{BD}{\sqrt{3}}\right)\right] \cdot BD = \frac{1}{6}(BD)^3 = 2025\ cm^3$. Therefore, $BD = \sqrt[3]{6 \times 2025\ cm^3} = \sqrt[3]{2 \times 3^5 \times 5^2\ cm^3} = 3\sqrt[3]{2 \times 3^2 \times 5^2}\ cm = \mathbf{3\sqrt[3]{450}}$ cm.

Target 1

The integers 1 through 10 sum to $\frac{10(10+1)}{2} = 55$. Dropping one value would yield a value for the sum in the interval 45 through 54, of which the only perfect square is $7^2 = 49$, so $55 - 49 = \mathbf{6}$.

Target 2



Because the trapezoid is isosceles, $\overline{AD} \cong \overline{BC}$. Because $\overline{DE}$ and $\overline{CF}$ are heights of the trapezoid, $\overline{DE} \cong \overline{CF}$. Therefore, by the Hypotenuse-Leg theorem, $\triangle DAE \cong \triangle CBF$, so corresponding sides $\overline{AE}$ and $\overline{BF}$ are congruent. As opposite sides of a rectangle, $EF = DC = 15$ in. $AE + BF = 21\ in - EF = 6$ in, so $AE = BF = 3$ in. Thus right triangles DAE and CBF are 3-4-5 triangles with scale factor 3 in and $AD = BC = 5$ in. Therefore, the perimeter is $21\ in + 15\ in + 2 \times 5\ in = \mathbf{46}$ in.

Target 3

Line 1 passes through $(2,\ 5)$ and $(0,\ 7)$, so its slope is $m_1 = \frac{7-5}{0-2} = -1$.

The slope of perpendicular line 2 is $m_2 = -\frac{1}{m_1} = -\frac{1}{-1} = 1$.

Therefore, line 2 passes through $(2,\ 5)$ and $(0,\ b_2)$ for $b_2 = 5 + (0-2)1 = \mathbf{3}$.

- [NOTE: There was some controversy over the form of the answer at some of the competitions. There are two different definitions of y-intercept in common use:
  (1) the y-value of a line or curve when $x = 0$;
  (2) the coordinates of a point, as an ordered pair, where a line or curve intersects the y-axis.
  In this question, the y-intercept would be stated as 3 per definition (1) and as $(0, 3)$ per definition (2). Some students wrote the latter as the answer—likely because a teacher taught them vehemently and vigorously that an intercept is always and only a point, which, therefore, *must* be expressed as an ordered pair with *both* components explicitly stated no matter what. Different mathematicians working in different branches of mathematics often find it convenient and useful to borrow a term from another branch but tweak the definition to suit their own special purposes; there is no authoritative organization in mathematics to remedy such differences. It is appropriate for a teacher to state the specific definitions that will be adhered to in class, likely subject to deduction of points if violated in classwork; however, students should also be made aware that there are people who use alternative definitions that are not wrong

Copyright MATHCOUNTS, Inc. 2025. All rights reserved.

outside that class, and students will likely encounter some of them, so a person needs to be flexible when reading mathematics material. In the case of this question, there are three very strong indicators—two within the question itself and one in the School Handbook—that the correct answer is 3, not (0, 3):

(1) The question states "the y-intercept of one of the lines is 7", thus providing an explicit example of what is being looked for, so follow the same pattern for the answer, even if that pattern does not match what you were taught. The instructions in a question prevail.

(2) With the one exception of monetary amounts, the default is that all answers are integers unless explicitly stated to the contrary (Express your answer in simplest radical form. Express your answer as a common fraction. …). An ordered pair of integers does not qualify as an integer, so if the answer was to be given as an ordered pair, then that would have been explicitly stated, but it was not.

(3), In this year's School Handbook on page 25 in the lower right corner, there is a table "Equation of a Line" and the second entry is "Slope-Intercept Form" with equation "$y = mx + b$", where "m = slope $b = y$-intercept", so that the real number b, thus not the coordinate $(0, b)$, is explicitly stated to be the y-intercept.

Therefore, the directive to scorers at the concerned competitions was to regard (0, 3) as an incorrect answer.

Target 4

The only ways to add distinct prime numbers to yield 10 are $2 + 3 + 5$ and $3 + 7$. For products of these sets of primes to be less than 100, we have $2 \times 3 \times 5 = 30$; $2^2 \times 3 \times 5 = 60$; $2 \times 3^2 \times 5 = 90$; $3 \times 7 = 21$; $3^2 \times 7 = 63$. The sum is $30 + 60 + 90 + 21 + 63 = \mathbf{264}$.

Target 5

By subtracting 2 from the various inequalities, $-6 \le x + 2 \le 6$ is equivalent to $-8 \le x \le 4$, which has $4 - (-8) + 1 = \mathbf{13}$ integer solutions.

Target 6

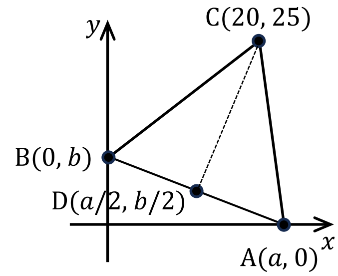
The two moduli, 5 and 7, are small, so brute force works faster than a high-powered technique like Chinese remainder theorem. To step through candidates faster, focus on the greater number 7. The least positive integer of the form $7k + 1$ is 7 (for $k = 0$). That is not a multiple of 5. Increment by 7: 8 (No); 15 (Yes). With the moduli 5 and 7 being relatively prime, the next qualifying values will occur in steps of $5 \times 7 = 35$, so the first three are 15; 50; 85. This is an arithmetic sequence of terms whose sum is the product of the count of terms times the median value: $3\times 50 = \mathbf{150}$.

Target 7

Designate the nth triangular number as T_n, with $T_n = 1 + 2 + 3 + \cdots + n = n(n + 1)/2$ for any positive integer n. Thus, T_8 is the sum of the integers 1 through 8. Subtracting T_6 means subtracting the integers 1 through 6, which are the first 6 terms of T_8, leaving only $7 + 8 = \mathbf{15}$.

Copyright MATHCOUNTS, Inc. 2025. All rights reserved.

Target 8



Because the triangle is specified to be equilateral, rotating line segment $\overline{BA}$ by $+60°$ about B would result in a match with $\overline{BC}$. Drop an altitude from C meeting $\overline{BA}$ at midpoint D. △CBD is a 30°-60°-90° triangle. Going from B to D involves moving $a/2$ right and $b/2$ down. DC = $\sqrt{3}$ BD and $\overline{DC} \perp \overline{BD}$, so the motion from D to C involves multiplying the motions from B to D by $\sqrt{3}$, and, because the slopes of two perpendicular lines are negative reciprocals of each other, swapping which value is horizontal and vertical, and negating one of the two motions so that we are moving up and right. Thus, point A would be transformed to $\left(\frac{1}{2}a + \frac{\sqrt{3}}{2}b,\ \frac{1}{2}b + \frac{\sqrt{3}}{2}a\right) = \text{C} = (20,\ 25)$. Therefore, we have a system of two linear equations in the two unknowns a and b.

$\frac{1}{2}a + \frac{\sqrt{3}}{2}b = 20$, which can be simplified a bit to Eqn ① $a + \sqrt{3}b = 40$.

$\frac{\sqrt{3}}{2}a + \frac{1}{2}b = 25$, which can be simplified a bit to Eqn ② $\sqrt{3}a + b = 50$.

$\sqrt{3}$① − ② yields $2b = 40\sqrt{3} - 50$, and $\sqrt{3}$② − ① yields $2a = 50\sqrt{3} - 40$. Therefore, $a = 25\sqrt{3} - 20$ and $b = 20\sqrt{3} - 25$. The area of an equilateral triangle with side length s is $\frac{\sqrt{3}s^2}{4}$.

Now, $s^2 = (\text{AB})^2 = a^2 + b^2 = \left(25\sqrt{3} - 20\right)^2 + \left(20\sqrt{3} - 25\right)^2$

$= \left(1875 - 1000\sqrt{3} + 400\right) + \left(1200 - 1000\sqrt{3} + 625\right) = 4100 - 2000\sqrt{3}$, so the triangular area is $\frac{\sqrt{3}(4100 - 2000\sqrt{3})}{4} = \frac{(4100\sqrt{3} - 6000)}{4} = 1025\sqrt{3} - 1500 = 275.352\ldots$, which rounds to the nearest hundredth as **275.35**.

Team 1

$A = 4P$; $100 = A + P = 5P$; $P = 100/5 = \mathbf{20}$.

Team 2

There is a total of $8 + 3 + 7 + 10 = 28$ students, of which 7 are wearing both long pants and a jacket, so the desired probability is $8/28 = \mathbf{2/7}$.

Team 3

Selling 60 kazoos requires buying $60/4 = 15$ packs at \$3 per pack for a total of \$45 spent. Then they are resold as 20 sets of 3 and \$2.85 per set for a total of \$57 revenue. The profit is \$57 − \$45 = \$**12** *or* \$**12.00**.

Team 4

Let d be the number of doughnuts bought by Ashton. Then $24 = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)d = d/3$, so $d = 3 \times 24 = \mathbf{72}$.

Team 5

There are 3 rows, 3 columns, and 2 diagonals for a total of 8 qualifying arrangements out of a total of $\binom{9}{3} = \frac{9\times8\times7}{3\times2\times1} = 12 \times 7 = 84$ possible arrangements (ordering of selection of the three squares is unimportant and not accounted for in both cases). Therefore, the probability is $\frac{8}{84} = \mathbf{\frac{2}{21}}$.

Copyright MATHCOUNTS, Inc. 2025. All rights reserved.

Team 6

$D = 2B = 3M$; $B = M + 8$. Therefore, $3M = D = 2B = 2(M + 8) = 2M + 16$. Subtracting $2M$ from both ends yields $M = 16$, $D = 3M = 3 \times 16 = \mathbf{48}$.

Team 7

Start with the greatest possible square and work downward decomposing squares into sums of smaller squares. For the following list, in the interest of space and readability, when a particular perfect square occurs at least 3 times, it is denoted as a product of the count times the square: [1] $4^2 + 1^2 + 1^2$; [2] $3^2 + 3^2$; [3] $3^2 + 2^2 + 2^2 + 1^2$; [4] $3^2 + 2^2 + 5(1^2)$; [5] $3^2 + 9(1^2)$; [6] $4(2^2) + 1^2 + 1^2$; [7] $3(2^2) + 6(1^2)$; [8] $2^2 + 2^2 + 10(1^2)$; [9] $2^2 + 14(1^2)$; [10] $18(1^2)$. There are **10** ways.

Team 8

The sum of the terms of a finite arithmetic sequence is the product of the number of terms times the average of the first and last terms. There are 100 terms, the first of which is given to be 32. We are also given that the second term is 35, so the common difference is $35 - 32 = 3$. The last term is term #100, which is 99 after term #1, so the last term is $32 + 99 \times 3 = 329$. Therefore, the sum is $100 \times \frac{32+329}{2} = 100 \times \frac{361}{2} = \mathbf{18\,050}$.

Team 9

Let p, n, and d be the number of pennies, nickels, and dimes, respectively. We are given $d = n + 3$, $n = p + 6$, so $d = p + 9$. The total number of coins is $p + n + d = p + (p + 6) + (p + 9) = 3p + 15$. We need the total value of the coins. The value of each penny, nickel, and dime is 1 ¢, 5 ¢, and 10 ¢, respectively. The total value is $p(1\text{¢}) + n(5\text{¢}) + d(10\text{¢}) = p(1\text{¢}) + (p + 6)(5\text{¢}) + (p + 9)(10\text{¢}) = 16p\text{¢} + 120\text{¢}$. The average coin value is 7 ¢, so the total value is $(3p + 15)(7\text{¢}) = 21p\text{¢} + 105\text{¢}$. The two expressions of the total value must be equal, so we have $16p\text{¢} + 120\text{¢} = 21p\text{¢} + 105\text{¢}$. Each term in the equation has a common factor ¢, so divide both sides of the equation by ¢. Subtracting $16p + 105$ from both sides yields $15 = 5p$, so $p = 15/5 = 3$ and $d = p + 9 = \mathbf{12}$.

Team 10

Set up an xy-coordinate plane with the x-axis along $\overline{AC}$ and the y-axis along $\overline{DB}$, so that the origin O is at the midpoint of $\overline{AC}$ and the four vertices in question are A(−12, 0), C(12, 0), B(0, 16), and D(0, 9)—with OD = 9 deriving from CDO being a 3-4-5 right triangle with scale factor 3, and OB = 16 deriving from CBO being a 3-4-5 right triangle with scale factor 4. The point of minimum sum must be on the y-axis, because moving to the right from the y-axis causes PA to increase slightly faster than PC decreases, plus PB and PD increase as well; symmetry says the case of P moving left of the y-axis behaves likewise. If P starts at O and moves toward D, each unit of movement causes PB + PD to decrease by 2 while PA + PC increases much less, for a net decrease in the sum. Once P passes D moving toward B, PB decreases at the same rate that PD increases, but PA and PC are increasing. Therefore, D is the optimum position, resulting in DA + DB + DC + DD = 15 + 7 + 15 + 0 = **37**.

Copyright MATHCOUNTS, Inc. 2025. All rights reserved.