

MATHCOUNTS® Problem of the Week Archive

Highly Composite Number – September 2, 2024

Problems & Solutions

A highly composite number (HCN) is a positive integer with more divisors than any smaller positive integer. Plato considered the HCN 5040 to be an ideal number for dividing things – such as citizens into cities or states – because it is divisible by all the integers from 1 to 12 excluding 11. However, there is an integer less than 5040 with the same property. What is this number?

The number that has this same property is the least common multiple (LCM) of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 12. Writing each number as their prime factorization, we have 1, 2, 3, 2^2 , 5, 2×3 , 7, 2^3 , 3^2 , 2×5 and $2^2 \times 3$. To find the LCM, we will multiply the highest power of each prime number together. The LCM is $2^3 \times 3^2 \times 5 \times 7 = 2520$.

However, 5040, since it is highly composite, has more factors than this lesser integer from the previous problem. How many more factors does 5040 have than the number you found as your answer to the first problem?

The prime factorization of 5040 is $2^4 \times 3^2 \times 5 \times 7$. Because a factor of 5040 can be divisible by 0, 1, 2, 3 or 4 twos (5 choices), 0, 1 or 2 threes (3 choices), and 0 or 1 fives and sevens (2 choices each), it follows that the number 5040 has $5 \times 3 \times 2 \times 2 = 60$ factors. Similarly, the prime factorization of 2520 is $2^3 \times 3^2 \times 5 \times 7$, and the number 2520 has $4 \times 3 \times 2 \times 2 = 48$ factors. This means 5040 has $60 - 48 = 12$ more factors.

Plato also noted that while 5040 is not divisible by 11, by simply subtracting a number X from it, you can get a relatively close number that is divisible by 11. What is the smallest value of X that, when subtracted from 5040, will result in a number divisible by 11?

If we divide 5040 by 11, we get 458 remainder 2. This means 5040 is two more than a multiple of 11. If we subtract 2 from 5040, we get 5038, which is 11×458 . The smallest value of X is 2.

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