MATHCOUNTS® Problem of the Week Archive

Last Week of School – June 17, 2024

Problems & Solutions

Many students are finishing up their school years about this time. The end of school often means students are scrambling to figure out what grades they need on their final exams to earn certain grades. Keenan is no different. His averages for the four quarters are 82%, 86%, 92% and 96% in his math class. He really wants to earn a 90% for the year. The end-of-year grade is determined using the four quarter grades and the final exam grade. If each of the quarter grades are weighted equally, and the final exam grade is one-third of his end-of-year grade, what is the lowest percent Keenan must earn on his final exam to earn at least a 90% when his end-of-year grade is calculated?

From the information, we can determine that the "weight" of his final exam grade is double the weight of each of the quarter grades. If we let x be his final exam grade, we see that his end-of-year grade is calculated in the following manner: (82 + 86 + 92 + 96 + x + x) ÷ 6. Since we want this to be equal to 90, we can set up the equation (82 + 86 + 92 + 96 + x + x) ÷ 6 = 90, which is 356 + 2x = 540. Subtracting 356 from both sides and then dividing by 2 gives us x = ***92****.*

The end of school also means field day! During field day, each class competes against the others in kickball, the three-legged relay, the obstacle course and basketball. Keenan's class has 16 students. Each student must participate in one event of field day. Two students will do the three-legged race, six students are needed for the obstacle course and eight students must play basketball. In how many ways could the 16 kids in Keenan's class be assigned to their field day events?

Writing out all of the options probably isn't the way we want to approach this. There are just too many! A good solution to this problem uses the concept of combinatorics. We know we need 2 of the 16 students to play kickball, so we need to know how many ways there are to choose 2 kids from a group of 16. (Notice the order in which we are choosing them doesn't matter.) We can find the number of ways by simplifying "16 choose 2," which is written $_{16}C_2$ *and simplified in the following manner: (16!) ÷ (2!(16 – 2)!) = (16!) ÷ (2!14!) = 120 ways. Now, we only have 14 kids left, and we have to choose 6 for the obstacle course. There are* $_{14}C_6 = 3003$ *ways to do this. We are then left with only eight kids, and they all must play basketball, so there is just one way to decide on the basketball players. Notice that each of the 120 possible kickball teams can be matched with each of the 3003 obstacle course teams, and then once those are matched, there will only be the one option of the remaining eight kids for the basketball team. This is a total of (120)(3003)(1) =* ***360,360*** *ways that the kids can be assigned to their field day events.*

The end of the school year also means that students are busy signing each other's yearbooks. Keenan's best friend is Jackson, and Jackson's favorite number is 13. Because of this, when he signs his friends' yearbooks, he starts on page 13 and writes a portion of his message on every page number that is a multiple of 13. If there are 87 pages in the yearbook, how many pages does Jackson write on?

Jackson is going to write on every page that is a multiple of 13. Dividing 87 by 13, we see that the quotient is 6 with a remainder of 9. This means that there are ***6*** *pages over which Jackson will write his message.*

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