MATHCOUNTS® Problem of the Week Archive

Yard Work – June 3, 2024

Problems & Solutions

Stephen spent this past weekend helping his mother get some yard work done. His first project was to build a rectangular planter with exterior dimensions of 6 inches by 12 inches by 18 inches. He originally planned to use wood that was 1 inch thick. However, he mistakenly bought wood that was 2 inches thick. How many fewer cubic inches of soil are required to fill the planter made with 2-inch-thick wood than would have been required to fill the planter had it been made with the 1-inch-thick wood?

Using the 2-inch-thick wood, the 6 inch by 12 inch by 18 inch planter's interior is 2 inches smaller in height and 4 inches smaller in both length and width. Thus, the planter made with 2-inch-thick wood has an internal volume of (6 – 2)(12 – 4)(18 – 4) = 448 cubic inches. Had the planter been made with 1-inch-thick wood, the height would have only been decreased by 1 inch and the length and width would have each decreased by 2 inches. Thus, the internal volume of the planter made with 1-inch-thick wood would have been (6 – 1)(12 – 2)(18 – 2) = 800 cubic inches. This is a difference of 800 – 448 = ***352*** *cubic inches.*

Next on Stephen's task list was spreading mulch in his mother's new garden. The garden is in the shape of an isosceles trapezoid. The large base is 10 feet long and the short base is 6 feet long. In the center of the garden is a tree with a diameter of 2 feet. If Stephen spreads mulch 2 inches thick, he would spread $8 – \pi/6$ cubic feet of mulch. What is the altitude, in feet, of the trapezoid?

The area of a trapezoid is equal to the average of the two bases times the height. Thus, the area of the garden is ((10 + 6)/2)(h) and to get the volume, we just multiply this area by the height (i.e. the depth of the mulch) which is 2 inches or 1/6 ft. So, the volume of the garden, when disregarding the tree, is (4/3)h. Now, to take the tree into consideration, we must subtract the volume of the tree trunk that will be buried in mulch, which is $1^2\pi(1/6) = \pi/6$ *cubic feet. Thus,* $(4/3)h – \pi/6 = 8 – \pi/6$*. Solving for h, we find that the height of the trapezoid must be* ***6*** *feet.*

Finally, Stephen's mother sent him to the store to buy flowers. She told him to by 6 pansies but did not specify which color(s). When he got to the store, he saw that they had four different colors in stock and that there were 20 plants of each color. How many distinct combinations of colors of pansies could he have bought?

If Stephen bought all one color of any pansy, there would be 4 ways he could do this. If Stephen bought 2 colors of pansies, he could buy 5 of one color and 1 of another color (4 × 3 = 12 combinations), 4 of one color and 2 of another (12 color combinations) or 3 of one color and 3 of another (6 combinations), which is a total of 12 + 12 + 6 = 30 combinations. If Stephen bought all four colors, we can envision the first four flowers selected being one of each color (let's say A, B, C and D), leaving two flowers that can be any combination of the 4 colors (AA, AB, AC, AD, BB, BC, BD, CC, CD, DD), thus making 10 possible color combinations. Finally, if he buys only 3 of the colors, let's assume the first three are one of each of three

colors (let's say A, B and C). This means that there are 4 ways he can choose the first three flowers and the last three flowers can be chosen 10 ways (AAA, AAB, AAC, ABB, ABC, ACC, BBB, BBC, BCC, CCC). Thus, there are (4)(10) = 40 ways three colors can be chosen. This gives a total of 4 + 30 + 10 + 40 = ***84*** *possible combinations.*

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