

MATHCOUNTS[®] Problem of the Week Archive

National High Five Day! – April 15, 2024

Problems & Solutions

The 3rd Thursday in April (April 18, 2024) is National High Five Day!

For National High Five Day, Ronnie's class decides that everyone in the class should exchange one high five with each other person in the class. If there are 20 people in Ronnie's class, how many high fives will be exchanged?

Each of the 20 people in Ronnie's class will high five 19 people (everyone but themselves). However, if we simply multiply 20×19 , we will have double-counted each of the high fives. For example, we will have counted when Ronnie high fives Ruth and when Ruth high fives Ronnie. This is actually the same high five, though, since each person in the class is exchanging only one high five with each other person. Thus, we'll need to account for this by dividing by 2, so there will have been $(20 \times 19)/2 = 190$ high fives.

At the end of Ronnie's lacrosse game, every player on his team high-fived every member of the opposing team once and said "good game." If each team had 15 members, how many high fives were exchanged?

If we look at team member #1 on Ronnie's lacrosse team, we know that he will give 15 high fives, one to each of the 15 members on the opposite team. Team member #2 on Ronnie's team will do the same, as will team member #3, and so on. Therefore, multiplying Ronnie's 15 team members by the 15 team members on the opposite team gives $15 \times 15 = 225$ high fives exchanged.

In an effort to spread "high-fiving cheer" Ronnie and Ruth decided they would each high five 2 people and ask each person they high five to high five 2 additional people (and ask each of them to do the same). After 4 cycles like this, how many high fives will be exchanged?

Let's track the high fives generated by Ronnie.

Ronnie high fives 2 people $\rightarrow 2$ high fives

Those 2 people high five 2 more people each $\rightarrow 2(2) = 4$ high fives

Those 4 people each high five 2 more people $\rightarrow 4(2) = 8$ high fives

Those 8 people each high five 2 more people $\rightarrow 8(2) = 16$ high fives

That's a total of $2 + 4 + 8 + 16 = 30$ high fives. Since Ruth did the same as Ronnie, together they would have generated $2(30) = 60$ high fives.

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