# MATHCOUNTS ${ }^{\circ}$ Problem of the Week Archive 

Valentine's Day Treats - February 12, 2024

## Problems

According to "A History of Valentine's Day Cards in America" by T.M. Wilson, in 1847 Esther Howland was the first to mass-produce Valentine's Day cards. She made them out of lace, paint and expensive paper, and each one was individually written by a skilled calligrapher. The average card sold for $\$ 7.50$ while others cost as much as $\$ 50$. If 10 cents in 1847 would be equivalent to $\$ 3.74$ today, how much would the average card and most expensive card have cost today?

Since we are given a ratio of 10 cents to $\$ 3.74$, we can set up 2 more ratios to find what $\$ 7.50$ and $\$ 50$ would convert to. Just remember to convert 10 cents to $\$ 0.10$ before beginning. An extended proportion would say that $0.10 / 3.74=7.50 / x=50 / y$. By cross-multiplying and solving for $x$ and $y$, we would see people were spending what would be equivalent to $\$ \mathbf{2 8 0 . 5 0}$ and $\$ \mathbf{1 , 8 7 0}$ for us today!

Kelly decided to celebrate Valentine's Day for an entire month. She started giving her Valentine 1 candy heart on Jan. $14^{\text {th }}, 2$ candy hearts on Jan. $15^{\text {th }}, 4$ candy hearts on Jan. $16^{\text {th }}$, and continued doubling the number of hearts each day until Feb. $14^{\text {th }}$. If 200 candy hearts come in a bag, how many bags of candy hearts would Kelly need just for Feb. $14^{\text {th }}$ ?

This is an exponential growth problem that shows how quickly an amount can grow when repeatedly doubled. The first day, she gave 1 candy. The second day, she gave $1 \times 2$ candies. The third day, she gave $1 \times 2 \times 2$ candies. She will keep multiplying by 2 until she gets to the $32^{\text {nd }}$ day. Therefore, the amount of candy she'll need just for Feb. $14^{\text {th }}$ is $1 \times 2^{31}$. This is $2,147,483,648$ pieces of candy. Dividing this by 200 for each bag of candy means she'll need 10,737,419 bags just to cover Valentine's Day!

For Valentine's Day, Kevin wanted to send Mary Beth 11 balloons, since that was her favorite number. In the store, plain-colored balloons cost $\$ 0.75$, multi-colored balloons cost $\$ 1.30$, and extra-large balloons cost $\$ 1.50$. How many different combinations of 11 balloons can Kevin buy if he only has $\$ 12.00$ ?

Making an orderly chart may be the best way to approach this problem. Start with buying as many of the extra-large balloons as possible, then methodically subtract an extra-large balloon, and so on. Though he can afford 8 extra-large balloons, he then could not afford 3 more to make the 11 balloons needed, So, the most extra-large balloons he can afford is 5 (\$7.50), leaving him just enough to buy 6 plain-colored balloons (\$4.50). Then, find possibilities with 4 extra-large balloons. Notice exchanging a multi-colored balloon for a plain-colored balloon raises the cost $\$ 0.55$. This may help when determining possibilities and finding patterns. Eventually, you will find 24 possible combinations!

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Join us next week, February 19 - 26, for Engineers Week 2024! Each weekday, we'll be posting an engineering activity or Problem of the Day to celebrate engineers. Learn more at www.mathcounts.org/eweek!

