# MATHCOUNTS ${ }^{\text {® }}$ Problem of the Week Archive I Have a Dream! - January 15, 2024 

## Problems \& Solutions

The third Monday of January is the holiday commemorating the birthday of Martin Luther King, Jr. The first time Martin Luther King Jr. Day was celebrated was in 1986. Try to solve these problems without using a calendar!

How many times has Martin Luther King Jr. Day been celebrated?
Martin Luther King Jr. Day has been celebrated 2024-1986=38 times, plus 1 (because we are including the Martin Luther King Jr. Day in 1986). So, Martin Luther King Jr. Day has been celebrated 39 times.

How many times since 1986 has Martin Luther King Jr. Day actually been celebrated on his birthday, January 15th?

Martin Luther King Jr. Day has been celebrated on his birthday, January 15 ${ }^{\text {th }, ~} 6$ times (in 1990, 1996, 2001, 2007, 2018, and 2024). There is a range of dates in which the third Monday of January can fall any day on or between the $15^{\text {th }}$ and the $21^{\text {st }}$ of January (see solution to next problem). If Martin Luther King Jr. Day is on January $15^{\text {th }}$ this year (2024), then we can find that it was on the $16^{\text {th }}$ last year, the $17^{\text {th }}$ the year before that, the $18^{\text {th }}$ the year before, and so on. You can continue in this way to solve this problem, but remember that you must account for leap years!

What is the probability Martin Luther King Jr.'s actual birth date falls on the third Monday in January?
Each day of the week must occur twice before a third Monday can occur. Therefore, 15 days must occur to arrive at a third Monday. The earliest date in January the nation can celebrate Martin Luther King Jr.'s birthday is January $15^{\text {th }}$. If January $1^{\text {st }}$ occurs on a Tuesday, the third Monday will occur two weeks and six days later, or 20 days later. The latest date in January the nation can celebrate Martin Luther King Jr's birthday is January $21^{\text {st. }}$. The third Monday can fall on any of the seven dates January $15^{\text {th }}$ through $21^{\text {st }}$. His actual birth date is on one of these days, January $15^{\text {th }}$. The probability that Martin Luther King Jr.'s actual birth date falls on the third Monday in January is 1/7.

Does a decade exist in which Martin Luther King Jr. Day will not be celebrated at least once on his birthday, January 15th? On which days could Martin Luther King Jr. Day be celebrated throughout this decade?

There is a pattern for the date of Martin Luther King Jr. Day. If we didn't need to worry about leap years, the pattern would look like this: $21,20,19,18,17,16,15,21,20,19,18,17,16,15,21,20$, and so on. Unfortunately, leap years do play a role, though, so every fourth year actually moves back two numbers instead of one to account for the leap day. So, for example, the pattern of dates for successive Martin Luther King Jr. Days might be: 21, 20, 19, 17, 16, 15, 21, 19, 18, 17, 16, 21, 20, 19, 18, 16, 15, 21, 20, and so on. Hence, the following string of 10 successive holidays could occur: 21, 19, 18, 17, 16, 21, 20, 19, 18, 16, which is a string of 10 days without a 15 among them. Hence, it could happen that there will be a decade without a Martin Luther King Jr. Day falling on King's actual birthday.

# MATHCOUNTS ${ }^{\text {P }}$ Problem of the Week Archive 

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