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2024 STATE COMPETITION Sprint Round Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

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DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

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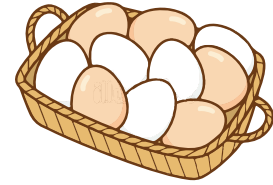
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1. _____ If ten more than twice a number equals 48, what is that number?

2. \$ _____ If 10 eggs cost \$0.75, how much will 32 eggs cost?



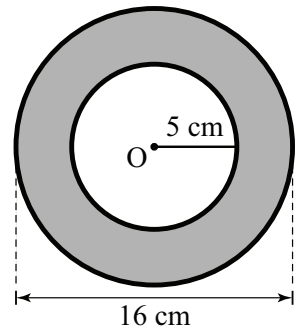
3. _____ The arithmetic mean of 3, -4 , 2, 0, 8 and x is 2. What is the value of x ?

4. _____ Suppose y is a two-digit multiple of both 4 and 6. The product the digits of y is 18. What is the value of y ?

5. _____ words Emily spelled each word on a 20-word spelling test. She earned 5 points for each word spelled correctly. Three points were subtracted for each word spelled incorrectly. Emily earned a score of 68. How many words did Emily spell incorrectly?

6. \$ _____ Carter has several tokens, each worth 25, 10 or 5 cents. The weight of a 25-cent token is 25 grams, the weight of a 10-cent token is 10 grams, and the weight of a 5-cent token is 5 grams. If the combined weight of all of Carter's tokens is 435 grams, what is the total value of his tokens?

7. _____ cm^2 Point O is the center of the two concentric circles shown. What is the area of the shaded region? Express your answer in terms of π .



8. _____ ounces A recipe calls for one 32-ounce jar of pickles, one packet of powdered drink mix, and $\frac{3}{4}$ cup of sugar. If the recipe amounts are increased proportionally so that 12 cups of sugar are used, how many ounces of pickles will be used?

9. _____ The mean of three positive integers a , b and c is 10, and the median of these integers is 5. What is the greatest possible value for c ?

10. _____ % There is a 50% chance of snow on Monday and a 60% chance of snow on Tuesday. Assuming weather on the two days is independent, what is the percentage chance that it snows on at least one of those two days?

11. _____ feet The distance an object falls from rest in a vacuum is proportional to the square of the time it has been falling. If a Holstein cow plushie takes 3 seconds to fall 144 feet in a vacuum, how far does the plushie fall in the first 2 seconds?

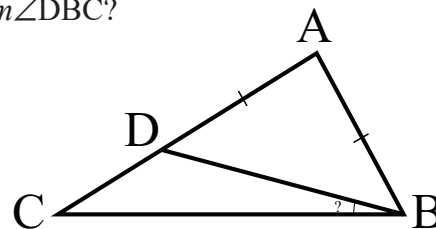
12. _____ kg Jameela weighed five giant pumpkins for the state fair. The weights, in kilograms, of all her pumpkins are integers. The median and mean weight of the five pumpkins are both 73 kg. If the weight of two of the pumpkins are 73 kg and 77 kg, what is the least possible value for the range of the weights?



13. _____ hours There is a unique time between 2 and 3 o'clock at which the minute and hour hands of a standard 12-hour clock form an angle of 180 degrees. How many hours after 2 o'clock is this time? Express your answer as a common fraction.

14. _____ round trips Robyn can travel from Lincoln to Omaha in 4 different ways and from Omaha to Des Moines in 3 different ways. How many different round trips can Robyn make from Lincoln to Des Moines that pass through Omaha in both directions?

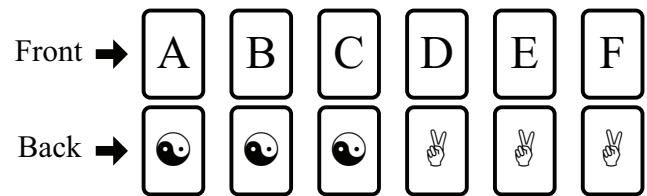
15. _____ degrees In triangle ABC, shown here, point D is on side AC such that $AB = AD$, and $m\angle ABC - m\angle ACB = 30$ degrees. What is $m\angle DBC$?



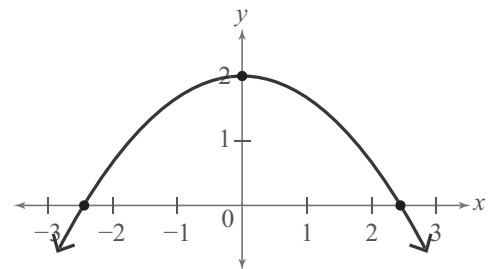
16. _____ If $x * y = \frac{x + y + xy}{x^2}$, what is the value of

$$\left[2 - \left(\frac{5}{2024} * 1 \right) \right] * \left[\left(\frac{5}{2024} * 1 \right) - 2 \right] ?$$

17. _____ The top row of this figure shows the letter printed on the front side of six cards, and the bottom row shows the image printed on the back side of the corresponding card. When Josie shuffles the cards and deals the first card front side down, the back shows ☯. What is the probability that the next card she deals has ✌ or A on either side? Express your answer as a common fraction.



18. _____ The figure shows the graph of $y = ax^2 + bx + c$ where a , b and c are constants. The function intersects the x -axis at $-\sqrt{6}$ and $\sqrt{6}$ and intersects the y -axis at 2. What is the value of $a + b + c$? Express your answer as a common fraction.



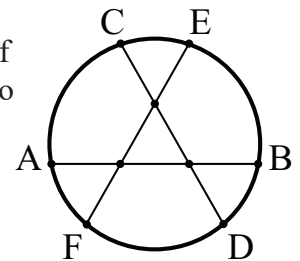
19. _____ If a , b and c are all positive integers greater than 1 for which $a + ab + abc = 415$, what is the value of c ?

20. _____ m^3 The height of a right circular cone is 12 meters and the surface area of the cone is $90\pi \text{ m}^2$. What is the volume of the cone? Express your answer in terms of π .

21. _____ What is the greatest prime factor of $9^6 - 1$ that does not divide $9^5 - 9^3 - 9^2 + 1$?

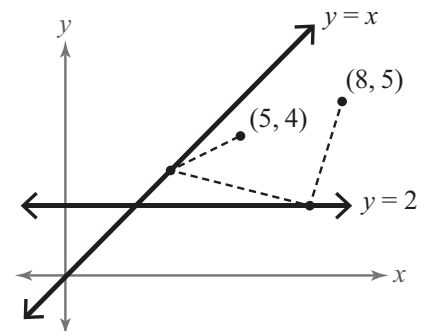
22. _____ Three fair nickels and two fair dimes are tossed. What is the probability that at least two heads are showing and that at least one of the heads appears on a dime? Express your answer as a common fraction.

23. _____ cm^2 In the circle shown, $AB = CD = EF = 3$ cm and the points of intersection among these three chords divide each chord into three congruent segments. What is the area of the circle? Express your answer as a common fraction in terms of π .



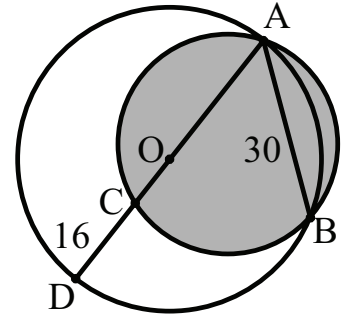
24. _____ Dennis rolls three fair six-sided dice, yielding values of a , b and c . What is the expected value of $|a - b| + |b - c| + |c - a|$? Express your answer as a common fraction.

25. _____ units A beetle starts at the point with coordinates $(8, 5)$, travels to a point on the line $y = 2$, then to some point on the line $y = x$, and then finally travels to the point with coordinates $(5, 4)$. One possible path that the beetle might follow is shown. What is the length of the shortest possible path that the beetle could take? Express your answer in simplest radical form.



26. _____ Positive numbers x , y and z satisfy the equations $xy(x+z)(y+z) = 165$, $xz(x+y)(z+y) = 88$ and $yz(z+x)(y+x) = 120$. What is the value of $x^2 + y^2 + z^2$? Express your answer as a common fraction.

27. _____ cm^2 In the figure shown, circle O has an area of $625\pi \text{ cm}^2$, $CD = 16 \text{ cm}$, and $AB = 30 \text{ cm}$. What is the area of the shaded circle, which intersects points A, B and C?



28. _____ factors How many positive integer factors does $\sqrt{\frac{2023! + 2019!}{2019!}} - 1$ have?

29. _____ units² In the coordinate plane, the two curves $x^4 + y^4 = 48$ and $xy = 2$ intersect in four points. What is the area of the quadrilateral whose vertices are these four points? Express your answer in simplest radical form.

30. _____ What is the greatest of the three prime number factors of $103^4 + 101^4 + 2^4$?

