

MATHCOUNTS[®] Problem of the Week Archive

Snowman Geometry – December 11, 2023

Problems & Solutions

For the following problems, assume that the snowballs are perfect spheres.

Chiquita and Rhonda are each building a snowman. Rhonda tells Chiquita that she is making each of her 3 balls of snow 20% smaller than the one below it. Rhonda starts by making the base ball with a diameter of 20 inches. If she reduces the diameter by 20% each time she makes a new ball, what is the diameter of the third ball she makes (the ball for the snowman's head)? Express your answer as a decimal to the nearest tenth.

The diameter of the first ball is 20 inches, and we can think of a 20% reduction in diameter as meaning the next ball will have a diameter of 80% of the previous ball's diameter. Thus, the diameter of the second ball is $0.8(20) = 16$ inches. This means that the diameter of the third ball is $0.8(16) = 12.8$ inches.

Chiquita planned to do the same thing as Rhonda, but she misunderstood what Rhonda meant when she said "...20% smaller than the one below it." Chiquita made her first ball the same size as Rhonda but then decreased the volume by 20% for each subsequent ball. How much taller is Chiquita's completed snowman than Rhonda's completed snowman? Express your answer as a decimal to the nearest tenth.

First, let's determine how big each of Chiquita's snowballs is. We know the first ball has a diameter of 20 inches, thus its volume is $(4/3)(20/2)^3\pi = 1333.3333\pi$ cubic inches. This means that the second ball has a volume of $0.8(1333.3333\pi) = 1066.6667\pi$ cubic inches, and the third ball has a volume of $0.8(1066.6667\pi) = 853.3334\pi$ cubic inches. The diameter of the third ball can be found by determining its radius ($V = (4/3)\pi r^3$) and then multiplying by 2. Thus, the diameter of the third ball is:

$$853.3334\pi = (4/3)\pi r^3$$

$$r = 8.617 \text{ inches}$$

$$d = 2(8.617) = 17.235 \text{ inches}$$

Following the same process, we find that the second ball has a diameter of 18.566 inches. We already knew the diameter of the first ball to be 20 inches. By adding 20, 18.566 and 17.235, we find the height of Chiquita's snowman to be 55.801 inches. We know the diameters of Rhonda's snowballs from the previous question, so we can quickly find her snowman's height to be $20 + 16 + 12.8 = 48.8$ inches.

Thus, the difference in height is $55.801 - 48.8 = 7.0$ inches, to the nearest tenth.

Rhonda's little brother, Benjamin, comes outside and decides to make a snowman out of cubes of snow instead of balls. If each of his three cubes has a side length equal to the diameter of the corresponding ball on Rhonda's snowman, how many more cubic inches of snow did Benjamin use than Rhonda? Express your answer as a decimal to the nearest tenth.

Again, based on the first question, we have the diameters of Rhonda's snowballs (20, 16, and 12.8 inches). Since the volume of a cube is (side length)³, we can just cube these lengths to determine the volume of snow Benjamin used.

$$20^3 = 8000$$

$$16^3 = 4096$$

$$12.8^3 = 2097.152$$

$$8000 + 4096 + 2097.152 = 14,193.152$$

Now, let's determine the volume of snow Rhonda used. Remember, for a sphere, $V = (4/3)\pi r^3$.

$$(4/3)10^3\pi = 1333.333\pi$$

$$(4/3)8^3\pi = 682.667\pi$$

$$(4/3)6.4^3\pi = 349.525\pi$$

$$1333.333\pi + 682.667\pi + 349.525\pi = 7431.5159\dots$$

Thus, Benjamin used $14,193.152 - 7431.516 = \mathbf{6761.6}$ cubic inches of snow, to the nearest tenth, more than Rhonda.

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