MATHCOUNTS® Problem of the Week Archive

Ghost of a Chance – October 30, 2023

Problems & Solutions

The letters of the words GHOST, GHOUL, TRICK, TREAT and CANDY were written on 25 index cards, one letter per card. If the cards are shuffled and one card is selected at random, what is the probability that the card contains one of the letters in the word GHOST?

*The 25 index cards contain the letters A, A, C, C, D, E, G, G, H, H, I, K, L, N, O, O, R, R, S, T, T, T, T, U and Y. Since two of the 25 cards contain the letter G, two contain the letter H, two contain the letter O, one contains the letter S and four contain the letter T, it follows that 2 + 2 + 2 + 1 + 4 = 11 of the 25 cards contain a letter in the word GHOST. Therefore, the probability of randomly selecting a card that contains a letter in the word GHOST is **11/25**.*

If all 25 cards are shuffled again, what is the probability of selecting, in any order and without replacement, five cards that contain the letters G, H, O, S and T, in the first five selections? Express your answer as a common fraction.

*Since two of the cards contain the letter G, the probability of randomly selecting one of those cards is 2/25. If a card with the letter G is selected, the probability of randomly selecting next a card with the letter H is 2/24. The probability that the next randomly selected card contains the letter O is 2/23, since two of the cards contain the letter O. Since only one card contains the letter S, the probability that the next randomly selected card contains the letter S is 1/22. There are four cards with the letter T, so the probability that the fifth randomly selected card contains the letter T is 4/21. Therefore, the probability of selecting the letters G, H, O, S and T (in that order) is (2/25) × (2/24) × (2/23) × (1/22) × (4/21) = 32/6,375,600 = 2/398,475. But we are asked to determine the probability of randomly selecting, in any order, cards containing these letters. Since any five cards can be selected in 5! = 120 different orders, the probability is 2/398,475 × 120 = 240/398,475 = **16/26,565**.*

*Another approach would be to consider all the ways that cards with the letters G, H, O, S and T can be selected and divide that by the total number of ways any five cards can randomly be selected. There are two ways to randomly select cards with each of the letters G, H and O. There is one way to randomly select a card with the letter S, and there are four ways to randomly select a card with the letter T. Therefore, the number of ways to select cards with the letters G, H, O, S and T is 2 × 2 × 2 × 1 × 4 = 32 ways. But cards with these five letters can be selected in 5! = 120 different orders, for a total of 32 × 120 = 3840 ways. The number of ways to randomly select five lettered cards is ${}_{25}P_5$ = 25!/20! = 25 × 24 × 23 × 22 × 21 = 6,375,600. Thus, the probability of randomly selecting cards with the letters G, H, O, S and T is 3840/6,375,600 = **16/26,565**.*

If all 25 cards are shuffled once more, what is the probability of selecting, in any order and without replacement, five cards that contain the letters T, R, I, C and K **or** five cards that contain the letters T, R, E, A and T, in the first five selections? Express your answer as a common fraction.

First, let's determine the probability of randomly selecting the letters T, R, I, C and K. Since four cards contain the letter T, the probability of randomly selecting one of those cards is 4/25. The probability of next randomly selecting one of the two cards containing the letter R is 2/24. The probability that the next card selected contains the letter I is 1/23. The probability that the next randomly selected card contains the letter C is 2/22, and the probability that the fifth card randomly selected contains the letter K is 1/21. Therefore, the probability of randomly selecting five cards containing the letter T, R, I, C and K (in that order) is (4/25) × (2/24) × (1/23) × (2/22) × (1/21) = 16/6,375,600 = 1/398,475. Since any five cards can be selected in 5! = 120 different orders, the probability of randomly selecting five cards with the letters T, R, I, C and K is 1/398,475 × 120 = 120/398,475 = 8/26,565.

Now, we can determine the probability of randomly selecting cards containing the letters T, R, E, A and T. Again, the chance of randomly selecting one of the four cards containing the letter T is 4/25, and the probability of next randomly selecting one of the two cards containing the letter R is 2/24. The probability that the next randomly selected card contains the letter E is 1/23, and the probability that the card randomly selected after that one contains the letter A is 2/22. Finally, the probability that the fifth randomly selected card also contains the letter T is 3/21, since one T has already been removed. Therefore, the probability of randomly selecting five cards containing the letters T, R, E, A and T (in that order) is (4/25) × (2/24) × (1/23) × (2/22) × (3/21) = 48/6,375,600 = 1/132,825. Five different cards can be selected in 5! = 120 different orders, but since the two cards containing the letter T are indistinguishable, we must divide this by 2! = 2. Therefore, these five cards can actually be selected in 120/2 = 60 different orders. So, the probability of randomly selecting five cards with the letters T, R, E, A and T is 1/132,825 × 60 = 60/132,825 = 12/26,565.

So, the probability of randomly selecting five cards that contain the letters T, R, I, C and K or five cards that contain the letters T, R, E, A and T is 8/26,565 + 12/26,565 = 20/26,565 = ***4/5313****.*

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If all 25 cards are shuffled once more, what is the probability of selecting, in any order and without replacement, five cards that contain the letters T, R, I, C and K **<u>or</u>** five cards that contain the letters T, R, E, A and T, in the first five selections? Express your answer as a common fraction.