# MATHCOUNTS ${ }^{\circ}$ Problem of the Week Archive <br> Graduation Gift Cards - May 22, 2023 

## Problems \& Solutions

Derrick received a total of 21 gift cards for graduation. He received gift cards in each of four denominations, $\$ 25, \$ 50, \$ 75$ and $\$ 100$. The number of $\$ 100$ gift cards Derrick got was twice the number of $\$ 75$ gift cards he received. The total value of the $\$ 50$ gift cards he got exceeded the total combined value of the $\$ 25, \$ 75$ and $\$ 100$ gift cards by $\$ 100$. How many $\$ 25$ gift cards did Derrick get for graduation?

From the information provided, we know that the number of $\$ 100$ gift cards Derrick received was twice the number of $\$ 75$ gift cards he received, but the total value of the $\$ 50$ gift cards exceeded the total value of all the other gift cards combined. Since he received a total of 21 gift cards for graduation, we can deduce that he most likely received more $\$ 50$ gift cards than he did any other denomination. But let's start by assuming he received the following:

| Value | Qty. | Total |
| :--- | :--- | :--- |
| $\$ 25$ | 9 | $\$ 225$ |
| $\$ 50$ | 9 | $\$ 450$ |
| $\$ 75$ | 1 | $\$ 75$ |
| $\$ 100$ | 2 | $\$ 200$ |

In this scenario, the total combined value of the $\$ 25$, $\$ 75$ and $\$ 100$ gift cards Derrick received is $225+75$ $+200=\$ 500$, which exceeds the $\$ 450$ worth of $\$ 50$ gift cards by $\$ 50$. That means the actual number of $\$ 25$ gift cards is fewer than 9 and, as we suspected, the number of $\$ 50$ gift cards is more than 9 . Let's try the following scenario:

| Value |  | Qty. Total |
| :---: | :---: | :---: |
| $\$ 25$ | 7 | $\$ 175$ |
| $\$ 50$ | 11 | $\$ 550$ |
| $\$ 75$ | 1 | $\$ 75$ |
| $\$ 100$ | 2 | $\$ 200$ |

In this scenario, the total combined value of the \$25, \$75 and \$100 gift cards Derrick received is $175+75$ $+200=\$ 450$, and the value of the $\$ 50$ gift cards is $\$ 550$. That's $550-450=\$ 100$ more in $\$ 50$ gift cards. That means the actual number of $\$ 25$ gift cards Derrick received for graduation was 7 gift cards.

With the gift card Derrick had left over from his recent birthday, the 22 gift cards have an average value of $\$ 50$. What is the value of the leftover birthday gift card?

From the previous problem, we know that the combined value of the 21 gift cards Derrick received for graduation was $450+550=\$ 1000$. When the 21 graduation gift cards are combined with the leftover birthday gift card, let's call it $g$, the average value is $\$ 50$. That means $(1000+g) / 22=50 \rightarrow 1000+g=$ $1100 \rightarrow g=100$. The value of the leftover birthday gift card, then, is $\$ 100$.

All 21 of the gift cards Derrick received for graduation were for stores at the local mall. He used five gift cards during the mall's Memorial Day Sale Spectacular. If Derrick used at least two different denominations of gift cards, what is the absolute difference between the greatest and least combined value of his remaining 16 gift cards?

The minimum Derrick could have spent with five gift cards in at least two different denominations would be $\$ 150$ if he used four $\$ 25$ gift cards and one $\$ 50$ gift card. This would leave him with the greatest combined value for the remaining 16 gift cards, $3(25)+10(50)+75+100(2)=\$ 850$. The maximum Derrick could have spent with five gift cards in at least two different denominations would be \$375 if he used both $\$ 100$ gift cards, the $\$ 75$ gift card and two $\$ 50$ gift cards. This would leave him with the least combined value for the remaining 16 gift cards, $7(25)+9(50)=\$ 625$. Therefore, the absolute difference between the greatest and least combined values for the remaining 16 gift cards is $850-625=\$ 225$.

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