

MATHCOUNTS[®] Problem of the Week Archive

Follow the Rules – May 8, 2023

Problems & Solutions

Define $x @ y$ as $(x^3 - y)/x$, for distinct positive integers x and y . What is the value of $5 @ 10$?

Evaluating $5 @ 10$ yields $(5^3 - 10)/5 = (125 - 10)/5 = 115/5 = 23$.

Define $a \# b$ as $a^2 - b^2 - ab$, for real numbers a and b . What is the value of $5 \# (4 \# 3)$?

Let's first evaluate the expression inside the parentheses. We have $4 \# 3 = 4^2 - 3^2 - 4(3) = 16 - 9 - 12 = -5$. We now evaluate $5 \# (-5)$ to get $5^2 - (-5)^2 - 5(-5) = 25 - 25 + 25 = 25$.

Define $m \$ n$ as $m^2 + 2m/n$ and define $m \& n$ as $(m^2 - n^2 + mn)/(2n)$. What is the value of $(4 \$ 2) \& 10$?

Again, let's first evaluate the expression inside the parentheses. We have $(4 \$ 2) = 4^2 + 2(4)/2 = 16 + 8/2 = 16 + 4 = 20$. We now evaluate $20 \& 10$ to get $[20^2 - 10^2 + 20(10)]/[2(10)] = (400 - 100 + 200)/20 = 500/20 = 25$.

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