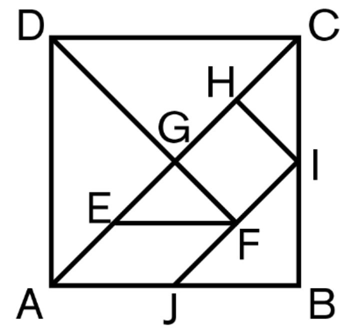
MATHCOUNTS® Problem of the Week Archive

Tangram Geometry – March 6, 2023

Problems & Solutions



The tangram is an ancient Chinese puzzle consisting of seven flat geometric shapes. These shapes, called tans, include one square, one parallelogram and five right isosceles triangles which are all similar. The seven tangram pieces here are arranged to form square ABCD of side length 8 units. Point G is the midpoint of diagonal AC and point E is the midpoint of segment AG. All the triangles are similar, and $\triangle ADG \cong \triangle CDG$ and $\triangle EGF \cong \triangle CHI$.

What is the area of quadrilateral AGFJ?

Let's first determine some of the missing measures in the figure. We know that $\triangle ADG$ is right and isosceles which makes it a 45-45-90 triangle. This fact, combined with the knowledge that DA = 8 units means that segments DG and AG each measure $8/\sqrt{2} = 4\sqrt{2}$ units. We are told that point E is the midpoint of segment AG, thus AE = EG = $(4\sqrt{2})/2 = 2\sqrt{2}$ units. Again, since $\triangle EGF$ is a 45-45-90 triangle, FG = $2\sqrt{2}$ units and EF = $(\sqrt{2})(2\sqrt{2}) = 4$ units. Now $\triangle EGF \cong \triangle CHI$, which means HI = CH = $2\sqrt{2}$ units and CI = 4 units. We also know that for the remaining side of square FGHI, IF = $2\sqrt{2}$ units. JF = AE = $2\sqrt{2}$ units and AJ = EF = 4 units for parallelogram AEFJ. Finally, JB = AB – AJ = 8 – 4 = 4 units, just as BI = BC – IC = 8 – 4 = 4 units. There are a number of ways to calculate the area of quadrilateral AGFJ, which happens to be a trapezoid. Here we will describe just one method. If we construct segment EJ such that it is perpendicular to both segments AJ and AE, we can see that quadrilateral AGFJ consists of 3 right, isosceles triangles which are congruent. Essentially the area of the trapezoid is 3 times the area of $\triangle EGF$. The area of $\triangle EGF$ is equal to $(1/2)(2\sqrt{2})(2\sqrt{2}) = 4$ units^2. Thus, the area of quadrilateral AGFJ is 3(4) = ***12*** *units^2.*

What is the ratio of the perimeter of parallelogram AEFJ to the perimeter of square FGHI? Express your answer as a common fraction in simplest radical form.

The perimeter of parallelogram AEFJ is $2(2\sqrt{2}) + 2(4) = 4\sqrt{2} + 8 = 4(\sqrt{2} + 2)$ units. The perimeter of square FGHI is $4(2\sqrt{2}) = 8\sqrt{2}$ units. The ratio of the perimeters is then $4(\sqrt{2} + 2)/(8\sqrt{2}) = (\sqrt{2} + 2)/(2\sqrt{2}) = 2(\sqrt{2} + 1)/4 =$ ***$(1 + \sqrt{2})/2$****.*

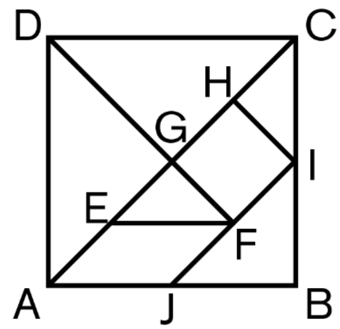
If triangle BIJ is rotated 360 degrees about segment BI, what would be the volume of the resulting solid? Express your answer as a common fraction in terms of π.

The solid formed by rotating $\triangle BIJ$ 360 degrees about segment BI is a cone with height and radius each measuring 4 units. So, the volume is $(1/3)\pi r^2 h = (1/3)\pi(4^2)(4) =$ ***$64\pi/3$*** *units^3.*

MATHCOUNTS® Problem of the Week Archive

Tangram Geometry – March 6, 2023

Problems



The tangram is an ancient Chinese puzzle consisting of seven flat geometric shapes. These shapes, called tans, include one square, one parallelogram and five right isosceles triangles which are all similar. The seven tangram pieces here are arranged to form square ABCD of side length 8 units. Point G is the midpoint of diagonal AC and point E is the midpoint of segment AG. All the triangles are similar, and $\triangle ADG \cong \triangle CDG$ and $\triangle EGF \cong \triangle CHI$.

What is the area of quadrilateral AGFJ?

What is the ratio of the perimeter of parallelogram AEFJ to the perimeter of square FGHI? Express your answer as a common fraction in simplest radical form.

If triangle BIJ is rotated 360 degrees about segment BI, what would be the volume of the resulting solid? Express your answer as a common fraction in terms of π.