

MATHCOUNTS[®] Problem of the Week Archive

Counting Cookies – December 26, 2022

Problems & Solutions

Santiago baked gingerbread cookies in the shapes of children. He gave each gingerbread child eyes made from chocolate covered candies in one of four different colors, a nose made from a butterscotch chip, toffee chip or peanut butter chip, and a mouth made from a piece of red licorice. Santiago then gave each gingerbread child buttons made from gum drops in one of four different flavors. For the final detail, he gave each gingerbread child a hat in either yellow or green. Based on these options, how many dozens of cookies did Santiago bake if each gingerbread cookie he made was uniquely decorated with a different combination of eyes, nose, mouth, buttons and hat?

We can use the Fundamental Counting Principle to determine the number of combinations of the various options for each gingerbread cookie. The first option is which of the 4 colors to use for the eyes. Then there are 3 options for the nose and only 1 option for the mouth. Next, the gum drop buttons can be one of 4 different flavors. Finally, each cookie has either a green hat or a yellow hat. That means there are $4 \times 3 \times 1 \times 4 \times 2 = 96$ different combinations of these characteristics. Therefore, the number of gingerbread cookies Santiago baked was $96/12 = 8$ dozens.

Suppose that instead of choosing one of the four different gum drop flavors for the buttons, Santiago decided that each gingerbread child would have three buttons, each made from a different flavored gum drop. If there are four different flavors of gum drops, how many different combinations of three gum drop buttons are possible?

Since there are four different flavors of gum drops from which to choose the three buttons, there are ${}_4C_3 = 4!/(3!1!) = 4$ different combinations of three buttons that Santiago could possibly choose. Another way to think about the number of combinations of 3 of the 4 different flavored gum drops is that there are 4 ways to select the 1 gum drop flavor that is not used for one of the three buttons.

How many different arrangements of three buttons can Santiago make using the combinations of three different flavored gum drops from the previous problem?

Each of the 4 combinations of three different flavored gum drops from the previous problem can be arranged in $3! = 6$ different ways. Therefore, the number of arrangements of three different flavored gum drops that Santiago can make is $4 \times 6 = 24$ arrangements. Another way to think about the number of arrangements of three different gum drops for the buttons is that there are 4 options for the first button, 3 for the second button and 2 for the third button. Using the Fundamental Counting Principle, that's a total of $4 \times 3 \times 2 = 24$ arrangements.

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