

# MATHCOUNTS<sup>®</sup> Problem of the Week Archive

## Counting Cookies – December 26, 2022

### Problems & Solutions

Santiago baked gingerbread cookies in the shapes of children. He gave each gingerbread child eyes made from chocolate covered candies in one of four different colors, a nose made from a butterscotch chip, toffee chip or peanut butter chip, and a mouth made from a piece of red licorice. Santiago then gave each gingerbread child buttons made from gum drops in one of four different flavors. For the final detail, he gave each gingerbread child a hat in either yellow or green. Based on these options, how many dozens of cookies did Santiago bake if each gingerbread cookie he made was uniquely decorated with a different combination of eyes, nose, mouth, buttons and hat?

*We can use the Fundamental Counting Principle to determine the number of combinations of the various options for each gingerbread cookie. The first option is which of the 4 colors to use for the eyes. Then there are 3 options for the nose and only 1 option for the mouth. Next, the gum drop buttons can be one of 4 different flavors. Finally, each cookie has either a green hat or a yellow hat. That means there are  $4 \times 3 \times 1 \times 4 \times 2 = 96$  different combinations of these characteristics. Therefore, the number of gingerbread cookies Santiago baked was  $96/12 = 8$  dozens.*

Suppose that instead of choosing one of the four different gum drop flavors for the buttons, Santiago decided that each gingerbread child would have three buttons, each made from a different flavored gum drop. If there are four different flavors of gum drops, how many different combinations of three gum drop buttons are possible?

*Since there are four different flavors of gum drops from which to choose the three buttons, there are  ${}_4C_3 = 4!/(3!1!) = 4$  different combinations of three buttons that Santiago could possibly choose. Another way to think about the number of combinations of 3 of the 4 different flavored gum drops is that there are 4 ways to select the 1 gum drop flavor that is not used for one of the three buttons.*

How many different arrangements of three buttons can Santiago make using the combinations of three different flavored gum drops from the previous problem?

*Each of the 4 combinations of three different flavored gum drops from the previous problem can be arranged in  $3! = 6$  different ways. Therefore, the number of arrangements of three different flavored gum drops that Santiago can make is  $4 \times 6 = 24$  arrangements. Another way to think about the number of arrangements of three different gum drops for the buttons is that there are 4 options for the first button, 3 for the second button and 2 for the third button. Using the Fundamental Counting Principle, that's a total of  $4 \times 3 \times 2 = 24$  arrangements.*

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### ***Problems***

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