MATHCOUNTS® Problem of the Week Archive

Game Show Probability – January 23, 2023

Problems & Solutions

Troy, Elliott and Shaun are three contestants on a game show. In one of the rounds of the game show, the 3 contestants have a chance to spin a wheel up to two times in a row to try to get the highest possible sum without going over 100. The wheel that they spin is divided into 21 equal portions, each containing one element from the set of numbers containing 1 and all the multiples of 5 less than or equal to 100.

Troy goes first. If Troy spins the wheel twice, getting a 15 and a 50, what is the probability that on Elliott's first spin, he gets a number larger than Troy's sum? Express your answer as a common fraction.

Troy's sum is 15 + 50 = 65. The possible spins greater than that are 70, 75, 80, 85, 90, 95, 100. The probability that Elliott spins one of those is 7/21 = ***1/3****.*

Elliott spins a 10 on his first spin. If Elliott spins the wheel a second time, what is the probability that the sum of his two spins will be a multiple of 4? Express your answer as a common fraction.

Since Elliott spun a 10 on his first spin, the only possible sums he can get will end in a 5 or a 0 or will be 11. We know that it is not possible for a multiple of 4 to end in 5. We also know that 11 is not a multiple of 4, thus we are only concerned with numbers that end in 0 that are also a multiple of 4. This tells us that the numbers we want are multiples of 4(5) = 20. Elliott could spin 10, 30, 50, 70 or 90 to end up with a sum that is a multiple of 4. That is a probability of ***5/21****.*

After Elliott's second spin, he ended up with a sum of 50. This means Troy was still in the lead. If Shaun spins a number greater than 65 on his first spin, he will not spin again. If Shaun spins a number less than or equal to 65, he will spin again. What is the probability that Shaun ends up being the winner? Express your answer as a common fraction.

The probability that Shaun wins on the first spin is 1/3, as it was for Elliott in the first problem. Now we need to figure out the probability of Shaun winning in two spins. Let's make a list. Notice that we only look at first spins up to 65 because if Shaun spun 70 or higher on his first spin, he wouldn't spin again.

1st spin	Winning 2nd spins	Number of combinations
1	65 through 95	7
5	65 through 95	7
10	60 through 90	7
15	55 through 85	7
20	50 through 80	7
25	45 through 75	7
30	40 through 70	7
35	35 through 65	7
40	30 through 60	7
45	25 through 55	7
50	20 through 50	7
55	15 through 45	7
60	10 through 40	7
65	1 through 35	8

That is a total of 7(13) + 8 = 99 two-spin combinations that would result in a win. That is a probability of 99/[(21)(21)] = 11/49. Thus, the probability that Shaun beats Troy is 1/3 + 11/49 = ***82/147****.*

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