MATHCOUNTS® Problem of the Week Archive

Solving Snowmen – January 9, 2023

Problems & Solutions

The weekend has just begun, and Derek wants to build a snowman either on Saturday or Sunday, but he can only build a snowman if it snows. There is a 1/2 probability that it snows on Saturday, and a 1/3 probability that it snows on Sunday. What is the probability that Derek will be able to build a snowman at least once over the weekend?

We can calculate the probability that Derek is <u>not</u> able to build a snowman and subtract it from 1 to get the probability that he is able to build one. He cannot build a snowman if it does not snow on both Saturday and Sunday. There is a 1 – 1/2 = 1/2 chance that is does not snow on Saturday and a 1 – 1/3 = 2/3 chance that is does not snow on Sunday. The probability that it does not snow at all over the weekend is (1/2) × (2/3) = 1/3. So, the probability that it does snow on at least one of the days, and therefore, the probability that Derek is able to build a snowman over the weekend is 1 – (1/3) = ***2/3****.*

Each section of the snowman's body begins as a sphere of radius 2 feet. However, due to the weight of snow, each section's radius shrinks by 0.1 foot for each pound of snow above it. If Derek's snowman has three sections, and they weigh 1 lb, 1.5 lb and 2 lb from top to bottom, how tall will his snowman be after all of the shrinking has occurred?

The bottom section has 1 + 1.5 = 2.5 pounds of snow above it, which means its radius will shrink by (0.1 ft/lb) × (2.5 lbs) = 0.25 ft. Its new radius is 2 – 0.25 = 1.75 feet.

The middle section has 1 pound of snow above it, which means its radius will shrink by (0.1 ft/lb) × (1 lb) = 0.1 ft. Its new radius is 2 – 0.1 = 1.9 feet.

The top section has no snow above it, so its radius will remain as 2 feet.

The total height of the snowman is two times the sum of the radii of each section, which yields 2(1.75 + 1.9 + 2) = ***11.3*** *feet.*

Derek's snowman has three sections, and he wants each section to have a prime number of buttons. If Derek has 10 buttons, in how many ways can he put them on his snowman? (Note that he does not necessarily need to use all of his buttons.)

Since there are a maximum of 10 buttons, the only possibilities for the number of buttons on each section of the snowman are primes less than 10: 2, 3, 5, 7.

If there are 7 buttons on any section, then the other two sections must have at least 2 buttons each, giving a minimum of 11 buttons. This is not possible, so every section of the snowman must have either 3 or 5 buttons.

If one of the sections has 5 buttons, then the other two sections can have either 2 buttons each or one can have 2 buttons and the other can have 3 buttons. Note that more than one section cannot have 5 buttons, as this would make the sum of buttons from all three sections greater than 10. This gives the combinations (5, 3, 2) and (5, 2, 2). There are 3! = 6 ways to permute the (5, 3, 2) button combination and 3 ways to permute the (5, 2, 2) button combination, giving a total of 6 + 3 = 9 combinations.

If each section has at most 3 buttons, then the maximum possible number of buttons that can be used is 3 buttons on each section, giving 9 buttons in total. Therefore, every combination of 2 buttons and 3 buttons is valid, as there will be a total sum less than 10. There are two possibilities (2 buttons or 3 buttons) for each section, and there are three sections, giving us 2^3 = 8 possibilities.

In total, this yields 9 + 8 = ***17*** *ways to add buttons to the snowman.*

Oh no! The sun is coming out, and Derek's snowman is melting! If snow melts at a rate of 1 cubic foot/hour, how long, in hours, will it take for Derek's snowman from problem 2 to fully melt? Express your answer as a decimal to the nearest tenth.

From problem 2, the radii of each section are 1.75, 1.9 and 2 feet. The volume of a sphere is $(4/3)\pi r^3$, so the total volume of the snowman is $(4/3)\pi(1.75)^3 + (4/3)\pi(1.9)^3 + (4/3)\pi(2)^3 \approx 84.7$ cubic feet. Therefore, the total time it will take the snowman to melt is ***84.7*** *hours.*

These problems were submitted by a MATHCOUNTS volunteer, Ishir Garg. Thank you, Ishir!

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