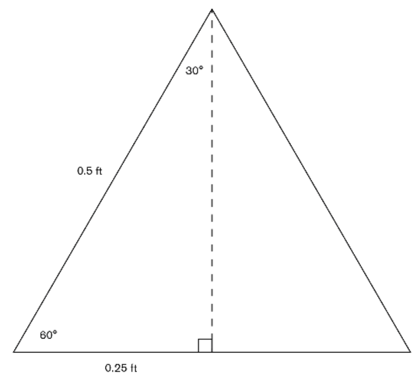
MATHCOUNTS® Problem of the Week Archive

Happy New Year! – January 2, 2023

Problems & Solutions

The Times Square Ball Drop in New York is an iconic event that takes place during New Years. The ball begins its "drop" at 11:59 p.m. ET on December 31 and ends its descent at 12 a.m. ET on January 1 to mark the beginning of the new year.

The surface of the ball is made up of 2,688 crystal triangles. If we approximate each triangle as equilateral with side length 0.5 foot, what is the total surface area of the ball? Express your answer to the nearest foot.



*Let's look at one of the equilateral triangles to find its area. We can divide the equilateral triangle into two 30-60-90 right triangles, as shown. Using properties of 30-60-90 right triangles, we know that the height of this triangle must be $0.25\sqrt{3}$. So, the area of one of these equilateral triangles is $(0.5)(0.25\sqrt{3})/2 = 0.1082531755$. With 2,688 of these triangles on the ball, the surface area of the ball is $2688 \times 0.1082531755 \approx$ **291** ft^2.*

While the ball drops only a few feet in reality, let's imagine the ball were to drop from its starting height all the way to the ground. The ball is suspended 141 feet in the air before being dropped. If it maintains a speed of exactly 3 ft/s while falling, how many seconds before midnight should the ball be dropped for it to reach the ground exactly at midnight?

Using distance = rate × time, we have $141 = 3 \times t$. Solving for t, we find that the ball would take $t = 47$ seconds to drop 141 feet at the given rate. So, the ball must be dropped ***47*** *seconds before midnight in order to reach the ground at exactly midnight.*

Mary wants to make a miniature version of the Times Square ball to hang in her room. Instead of crystal triangles, she'll glue small light bulbs onto a small foam sphere to recreate the sparkling effect of the crystals in a ball shape. Mary has some blue light bulbs, some white light bulbs, and some yellow light bulbs. If the foam sphere has room for 10 light bulbs and Mary wants at least one bulb of each color, how many different color *combinations* of lights on the ball are possible?

We know that there must be at least one bulb of each color, so no color will have 0 bulbs. Let's start with the situation in which there are 8 bulbs of one color and 1 bulb of each of the other two colors. There would be 3 possible such scenarios: (8, 1, 1), (1, 8, 1), (1, 1, 8), where the values represent the numbers of bulbs of each color in the order (blue, white, yellow). Next, there is the situation in which there are 7 bulbs of one color, 2 bulbs of another color, and 1 bulb of the third color. Here, there are 6 such scenarios: (7, 2, 1), (7, 1, 2), (1, 7, 2), (1, 2, 7), (2, 1, 7), (2, 7, 1). With 6 bulbs of one color, 3 bulbs of another color, and 1 bulb of the third color, there are also 6 possibilities. With 6 bulbs of one color and 2 bulbs of each of the other two colors, there are 3 possibilities. With 5 bulbs of one color, 4 bulbs of *another color, and 1 bulb of the third color, there are* <u>*6*</u> *possibilities. With 5 bulbs of one color, 3 bulbs of another color, and 2 bulbs of the third color, there are* <u>*6*</u> *possibilities. With 4 bulbs of one color and 3 bulbs of each of the other two colors, there are* <u>*3*</u> *possibilities. Finally, with 2 bulbs of one color and 4 bulbs of each of the other two colors, there are* <u>*3*</u> *possibilities. Therefore, there are 3 + 6 + 6 + 3 + 6 + 6 + 3 + 3 =* ***36*** *different color combinations of lights that could be on Mary's ball.*

These problems were submitted by a MATHCOUNTS volunteer, Ishir Garg. Thank you, Ishir!

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